Classical Mechanics

Phys105A, Winter 2007

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Formalities

- Latest news and course slides always found on the Phys105A site at http://www.cs.ucsb.edu/~vandam/…
- Homework 1 has been posted.
  It is due Monday January 22, 11:30 am.
- You have to hand in two separate sets of answers so that the two TAs can grade the different questions.
- Questions?

Cartesian Coordinates

- Rewriting the law of acceleration as **F** = m d²**r**/dt² in Cartesian (x,y,z) coordinates gives (of course):

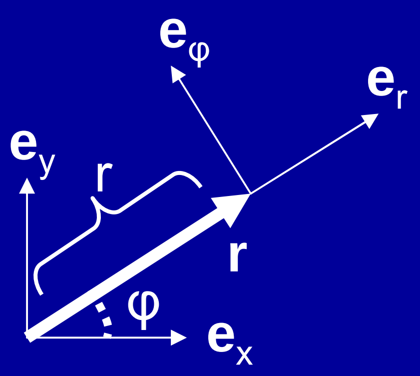
$$\mathbf{F} = m\ddot{\mathbf{r}} \quad \Leftrightarrow \quad \begin{cases} F_x = m\ddot{x} \\ F_y = m\ddot{y} \\ F_z = m\ddot{z} \end{cases}$$

- This allows us to decompose our calculations into three independent, 1 dimensional problems.

2D-Polar Coordinates

- Many 2 dimensional systems have symmetries that are best captured by *polar coordinates*:

$$\left.\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned} r &= \sqrt{x^2+y^2} \\ \varphi &= \arctan(y/x) \end{aligned}\right.$$



For a position **r**, we can define the corresponding coordinate system with $\mathbf{e}_r = \mathbf{r}/|\mathbf{r}|$ and $\mathbf{e}_\varphi \perp \mathbf{e}_r$.

As **r** changes in time, we can change our coordinates system such that always $\mathbf{r} = r(t)\,\mathbf{e}_r$, which is handy.

d/dt in Polar Coordinates

- For a particle with time dependent position **r** (with r,φ) we want to know its velocity d**r**/dt:
  - Observe that $d\mathbf{e}_r/dt = d\varphi/dt\ \mathbf{e}_\varphi$.
  - Hence we have: $\mathbf{v} = d\mathbf{r}/dt = dr/dt\ \mathbf{e}_r + r\ d\varphi/dt\ \mathbf{e}_\varphi$.

- We also want to know its acceleration d**v**/dt:
  - Observe that $d\mathbf{e}_\varphi/dt = -d\varphi/dt\ \mathbf{e}_r$
  - Hence we have for $d^2\mathbf{r}/dt^2 = d\mathbf{v}/dt$:

$$\mathbf{a} = [d^2r/dt^2 - r\ (d\varphi/dt)^2]\ \mathbf{e}_r + [r\ d^2\varphi/dt^2 + 2\ dr/dt\ d\varphi/dt]\ \mathbf{e}_\varphi.$$

Newton's Laws (Polar version)

- Newton's second law thus becomes

$$\mathbf{F} = m\mathbf{a} \quad \Leftrightarrow \quad \begin{cases} F_r = m(\ddot{r} - r\dot{\varphi}^2) \\ F_\varphi = m(r\ddot{\varphi} + 2\dot{r}\dot{\varphi}) \end{cases}$$

Despite its messy appearance, this rephrasing can be very useful as we often deal with situations where r stays constant, giving (cf. Example 1.2):

$$\mathbf{F} = m\mathbf{a} \quad \Leftrightarrow \quad \begin{cases} F_r = -mr\dot{\varphi}^2 \\ F_\varphi = mr\ddot{\varphi} \end{cases}$$

More Non-Cartesian Systems

- If an additional z-coordinate if required, we use *cylindrical (polar) coordinates* (ρ,φ,z), see p. 34.

- For spherical symmetries we use *spherical coordinates* (r,θ,φ), which will be dealt with in Chapter 4.

Chapter 2: Projectiles and Charged Particles

Modeling (Air) Resistance

- Realistically, a particle moving with through a medium will experience a *drag* (resistive force) **f**, which depends on the speed **v** of the particle.
- Ignoring the possibility of lift and other sideways forces, the direction will be opposite the speed: $\mathbf{f}/|\mathbf{f}| = -\mathbf{v}/|\mathbf{v}|$.
- Furthermore we assume that the magnitude of the drag is determined solely by v as well. Hence $\mathbf{f} = f(v)\ \mathbf{e}_v$.
- A reasonable approximation can be made by assuming that f is a quadratic function in v and f(0)=0.
- In sum: $f(v) = bv + cv^2 = f_{lin} + f_{quad}$.

Linear Air Resistance

- Consider a flying projectile of mass m for which the quadratic drag force is negligible: $\mathbf{f} = -b\,\mathbf{v}$.
- Summing the forces thus gives $\mathbf{F} = m\mathbf{g} - b\mathbf{v} = m\, d\mathbf{r}^2/dt^2$.
- With $\mathbf{r}(0) = \mathbf{0}$ and $\mathbf{v}(0) = \mathbf{v}_0$ we expect something like: