Particle Dynamics

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Overview

- One Lousy Particle
- Particle Systems
- Forces: gravity, springs …
- Implementation and Interaction
- Simple collisions

A Newtonian Particle

- Differential equation: f = ma
- Forces can depend on:
  – Position, Velocity, Time

$$\ddot{\mathbf{x}} = \frac{\mathbf{f}(\mathrm{x}, \dot{\mathrm{x}}, \mathrm{t})}{\mathrm{m}}$$

Second Order Equations

$$\ddot{\mathbf{x}} = \frac{\mathbf{f}(\mathrm{x}, \dot{\mathrm{x}}, \mathrm{t})}{\mathrm{m}}$$

Not in our standard form because it has 2nd derivatives

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{v} \\ \dot{\mathbf{v}} = \mathbf{f}/\mathrm{m} \end{cases}$$

Add a new variable, **v,** to get a pair of coupled 1st order equations.

Phase Space

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{v} \end{bmatrix}$$

Concatenate **x** and **v** to make a 6-vector: *Position in Phase Space.*

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix}$$

Velocity in Phase Space: another 6-vector.

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{f}/m \end{bmatrix}$$

A vanilla 1st-order differential equation.

Particle Structure

x — Position

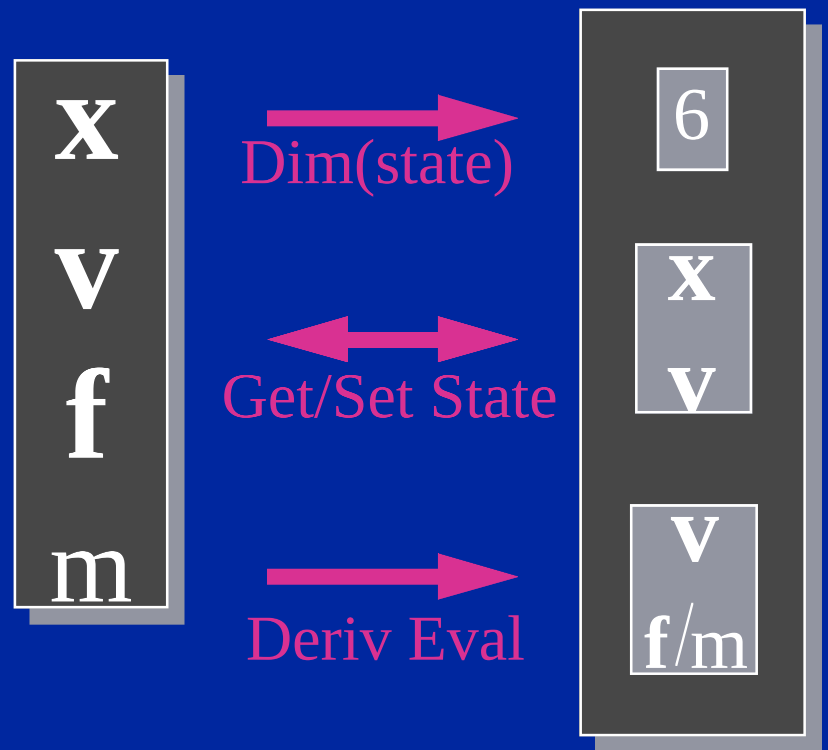
v — Velocity

f — Force Accumulator

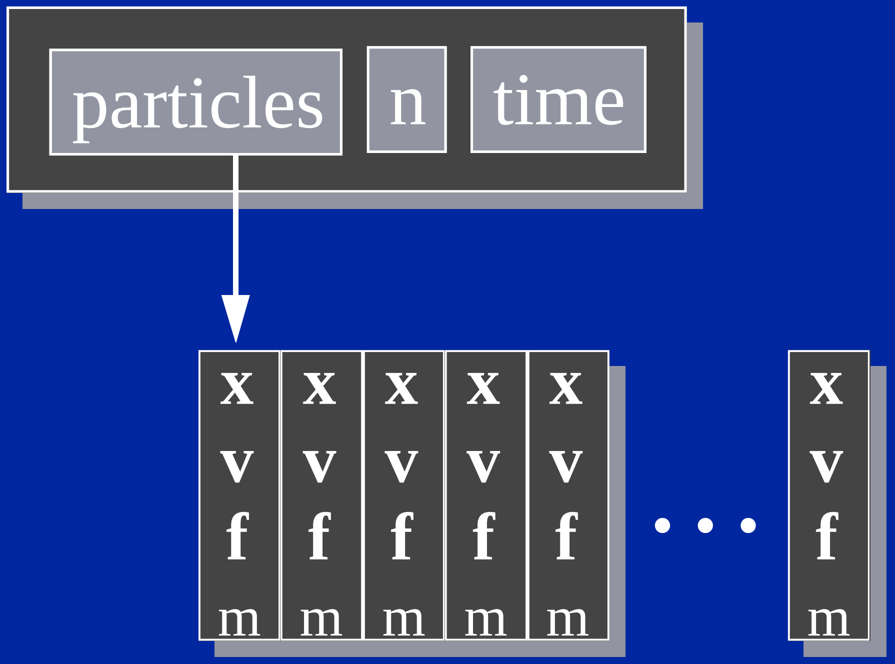
m — mass

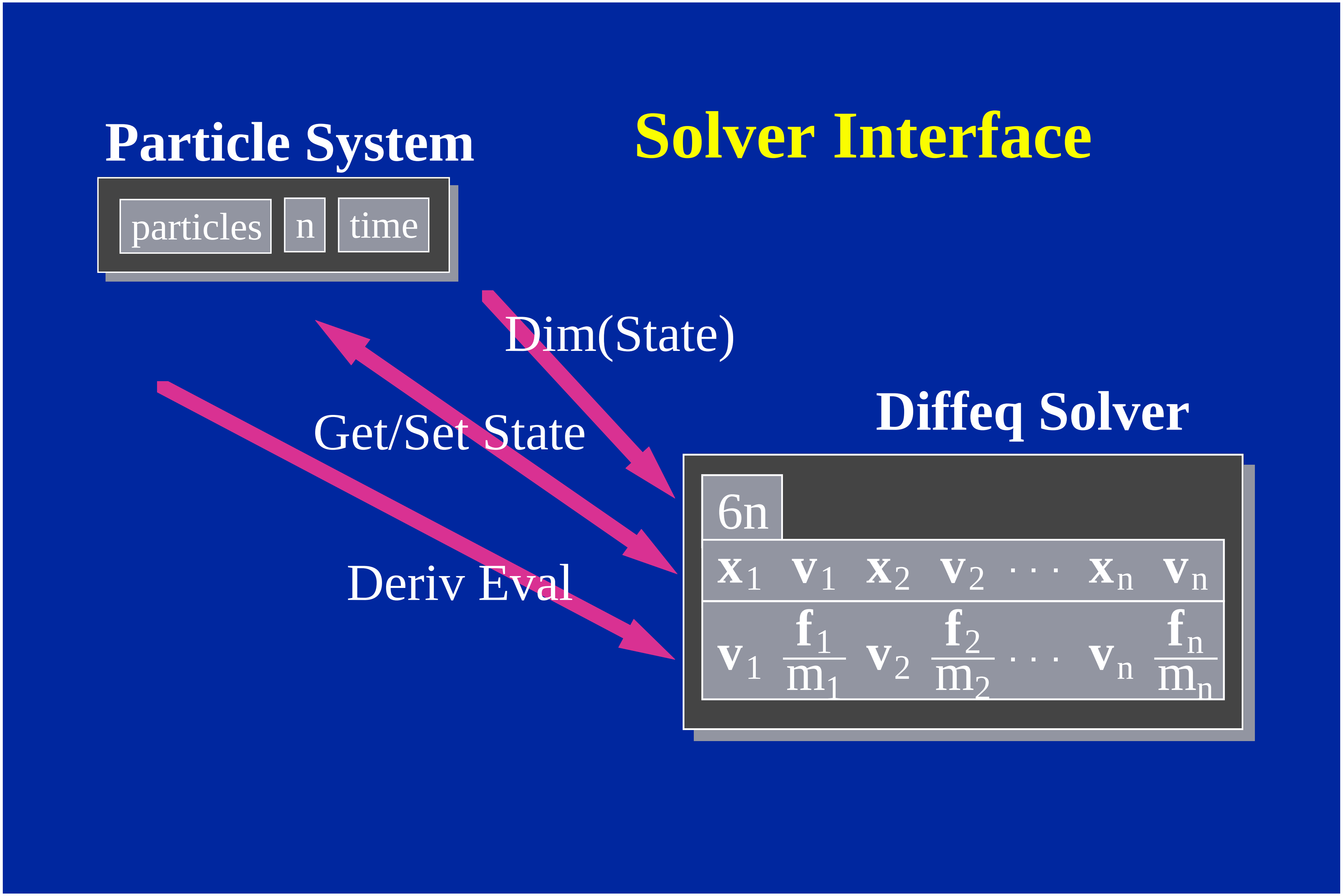
Position in Phase Space

Solver Interface



Particle Systems




Solver Interface
Particle System
particles
n
time
Dim(State)
Get/Set State
Deriv Eval
Diffeq Solver
6n
$\mathbf{x}_1$ $\mathbf{v}_1$ $\mathbf{x}_2$ $\mathbf{v}_2$ $\cdots$ $\mathbf{x}_n$ $\mathbf{v}_n$
$\mathbf{v}_1$ $\frac{\mathbf{f}_1}{m_1}$ $\mathbf{v}_2$ $\frac{\mathbf{f}_2}{m_2}$ $\cdots$ $\mathbf{v}_n$ $\frac{\mathbf{f}_n}{m_n}$

Deriv Eval Loop

- **Clear forces**
  - Loop over particles, zero force accumulators.
- **Calculate forces**
  - Sum all forces into accumulators.
- **Gather**
  - Loop over particles, copying **v** and **f**/m into destination array.

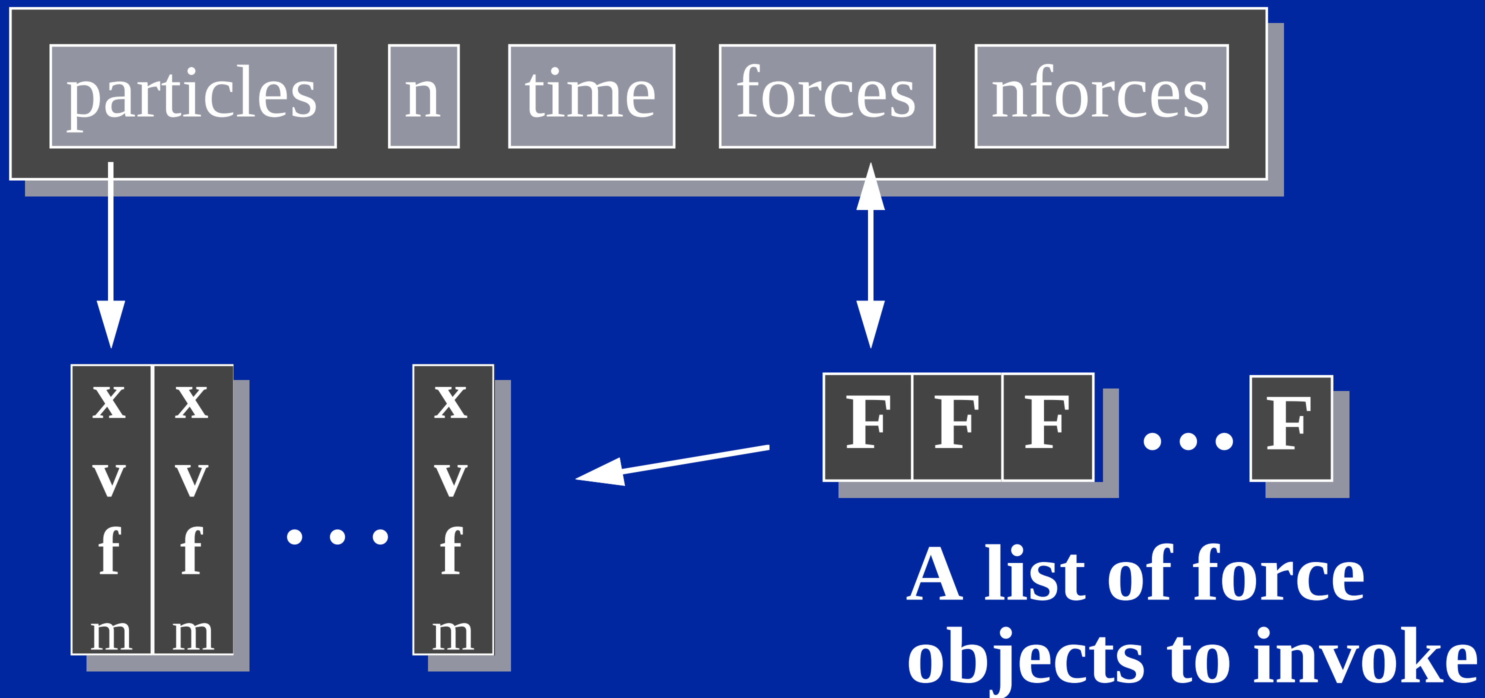
Forces

- **Constant** — **gravity**
- **Position/time dependent** — **force fields**
- **Velocity-Dependent** — **drag**
- **n-ary** — **springs**

Force Structures

- **Unlike particles, forces are heterogeneous.**
- **Force Objects:**
  - black boxes
  - point to the particles they influence
  - add in their own forces (type dependent)
- **Global force calculation:**
  - loop, invoking force objects

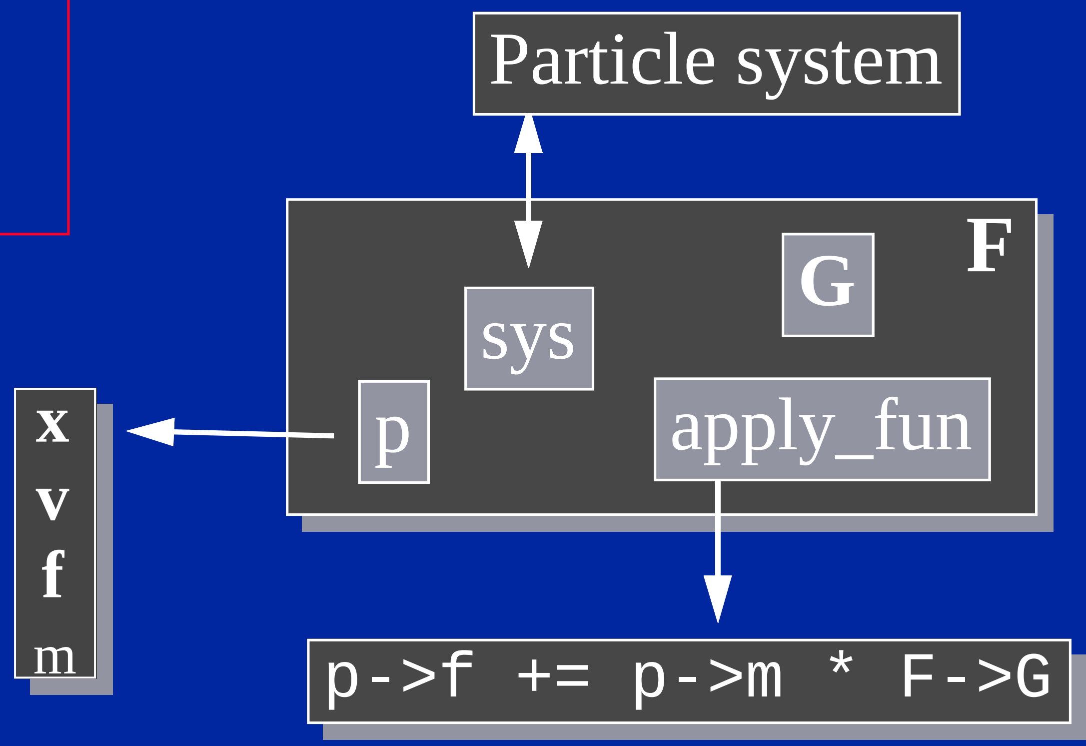
Particle Systems, with forces



Gravity

Force Law:

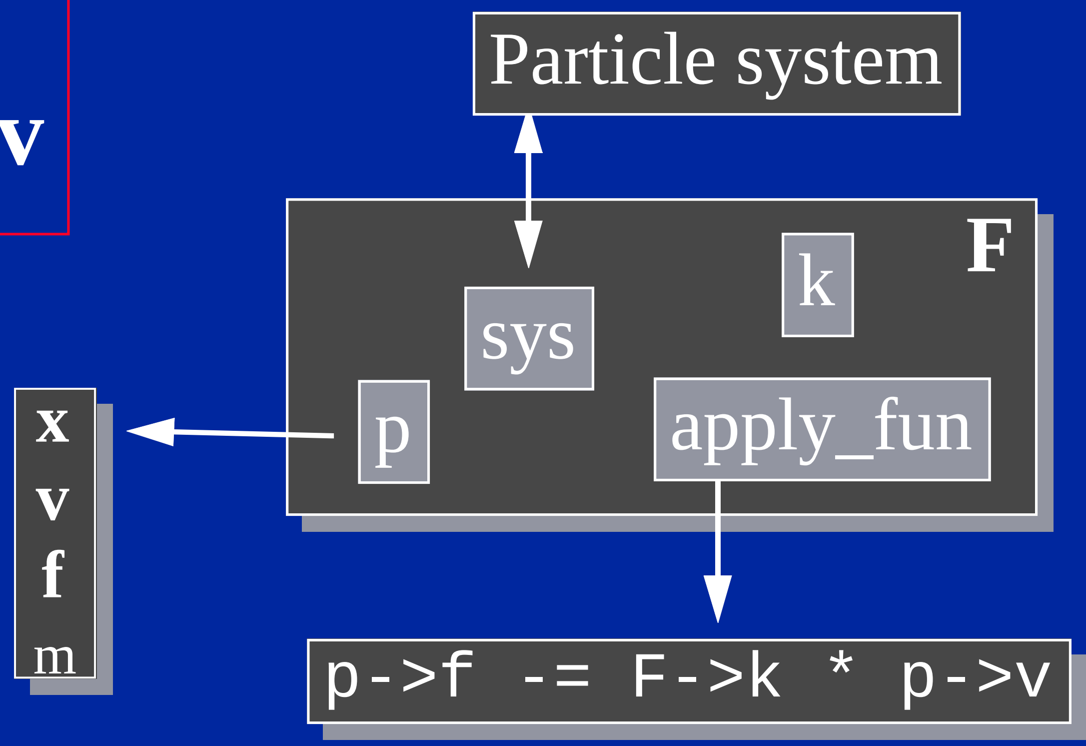
$$\mathbf{f}_{grav} = m\mathbf{G}$$



Viscous Drag

Force Law:

$$\mathbf{f}_{drag} = -k_{drag}\mathbf{v}$$

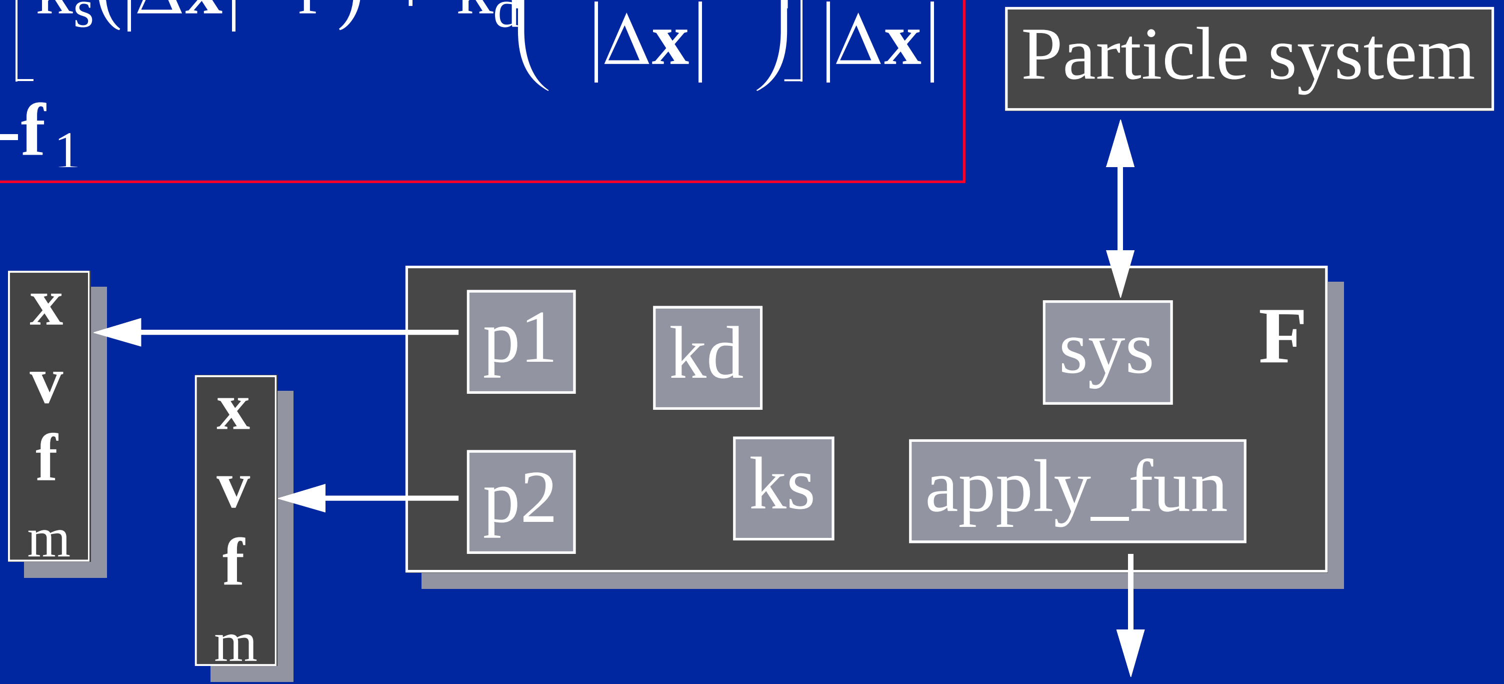


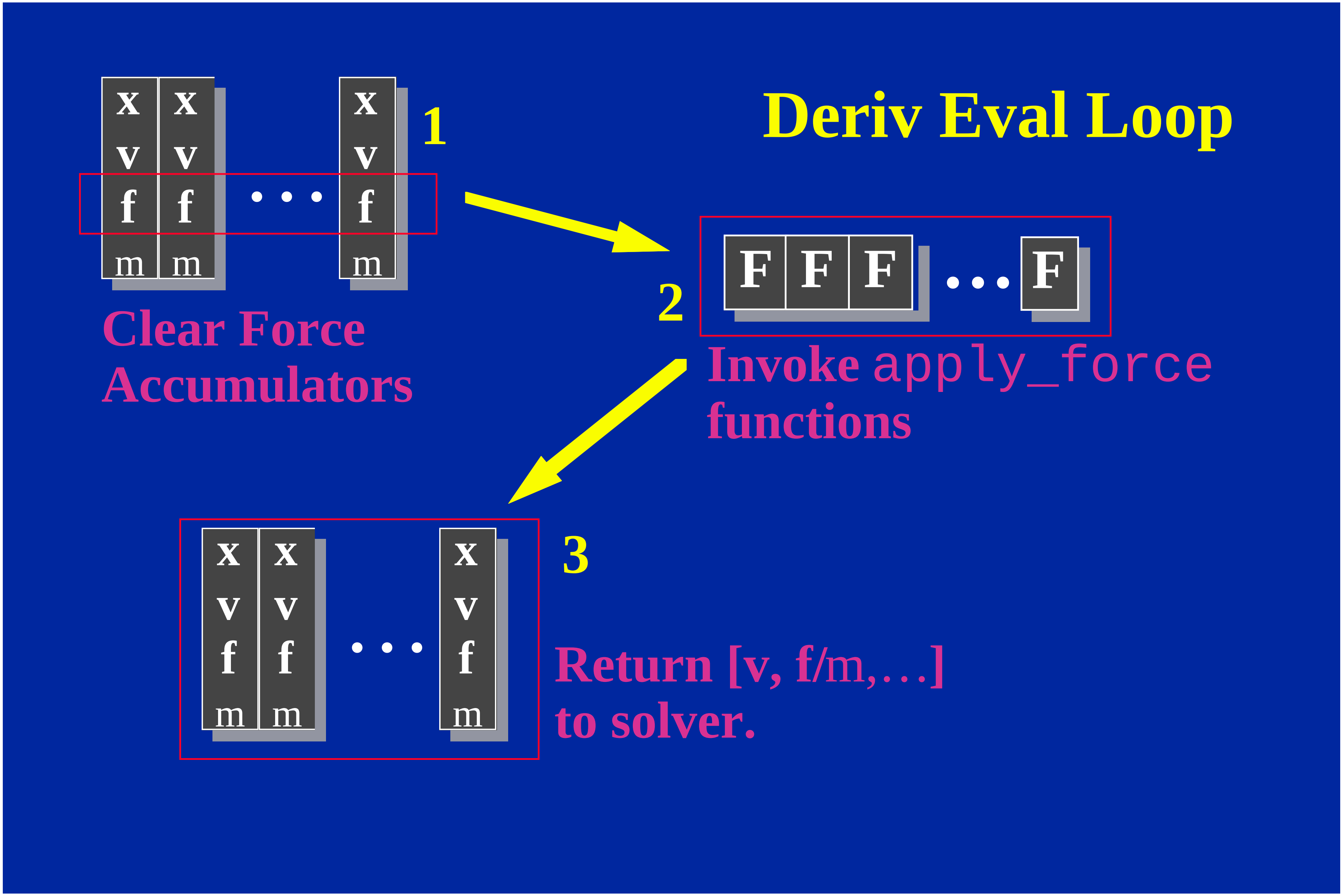
Damped Spring

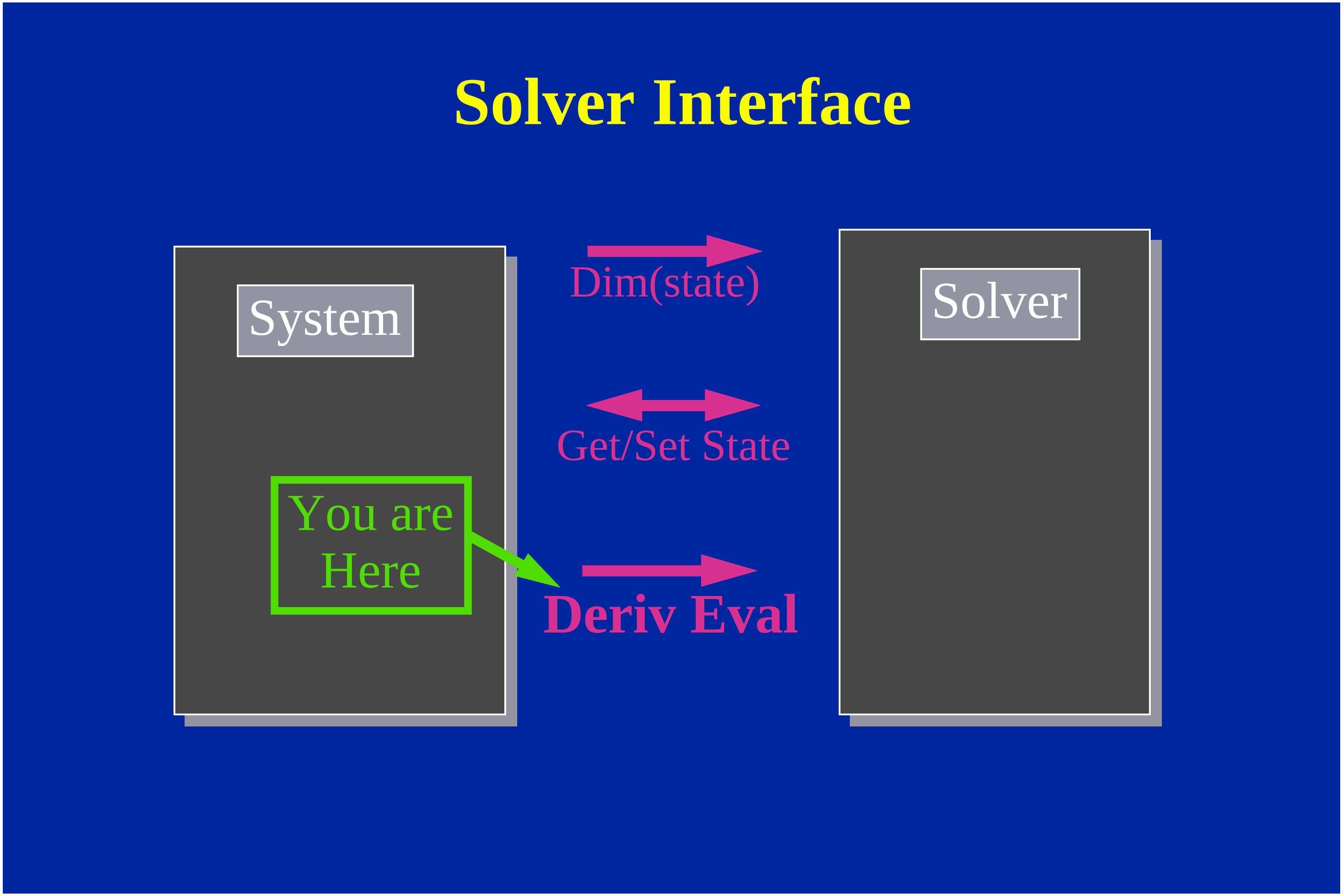
Force Law:

$$\mathbf{f}_1 = -\left[k_s\left(|\Delta\mathbf{x}| - r\right) + k_d\left(\frac{\Delta\mathbf{v}\cdot\Delta\mathbf{x}}{|\Delta\mathbf{x}|}\right)\right]\frac{\Delta\mathbf{x}}{|\Delta\mathbf{x}|}$$

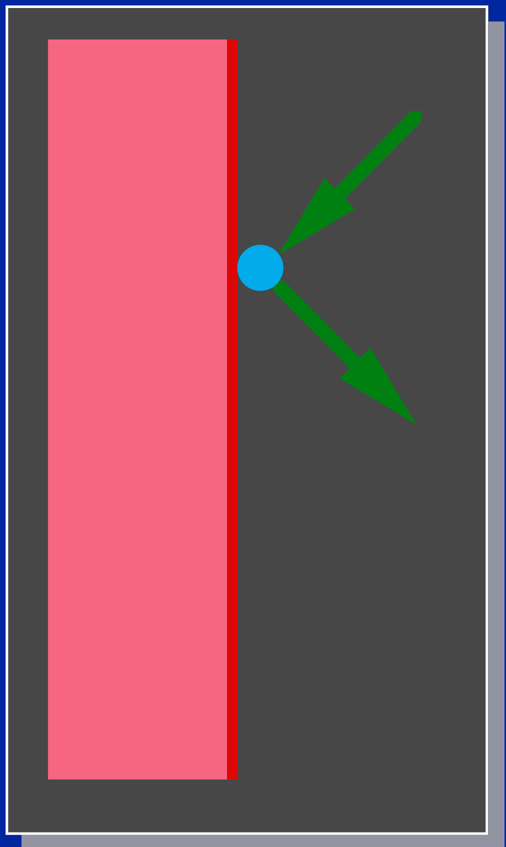
$$\mathbf{f}_2 = -\mathbf{f}_1$$




Deriv Eval Loop
x v f m
x v f m
x v f m
1
Clear Force Accumulators
F F F F
2
Invoke apply_force functions
x v f m
x v f m
x v f m
3
Return [v, f/m,…] to solver.

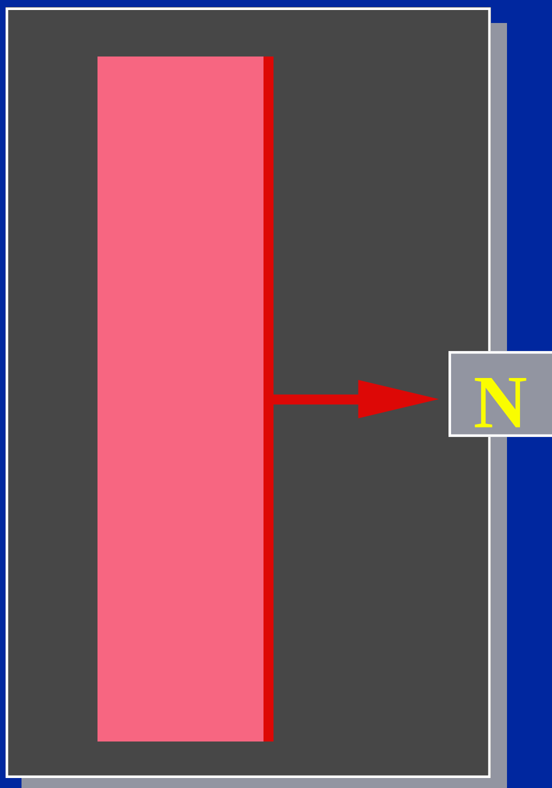

Solver Interface
System
Solver
Dim(state)
Get/Set State
You are Here
Deriv Eval

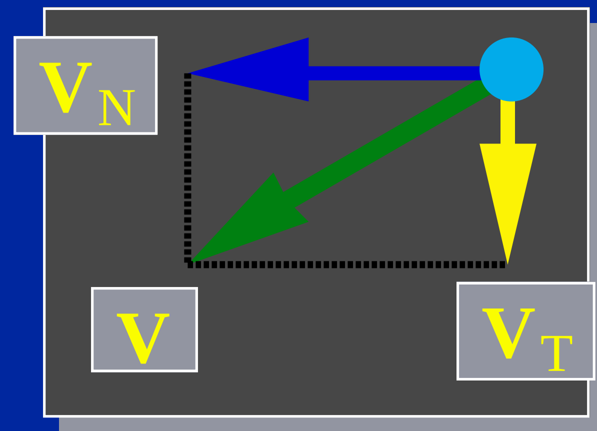
Bouncing off the Walls



- Later: rigid body collision and contact.
- For now, just simple point-plane collisions.
- Add-ons for a particle simulator.

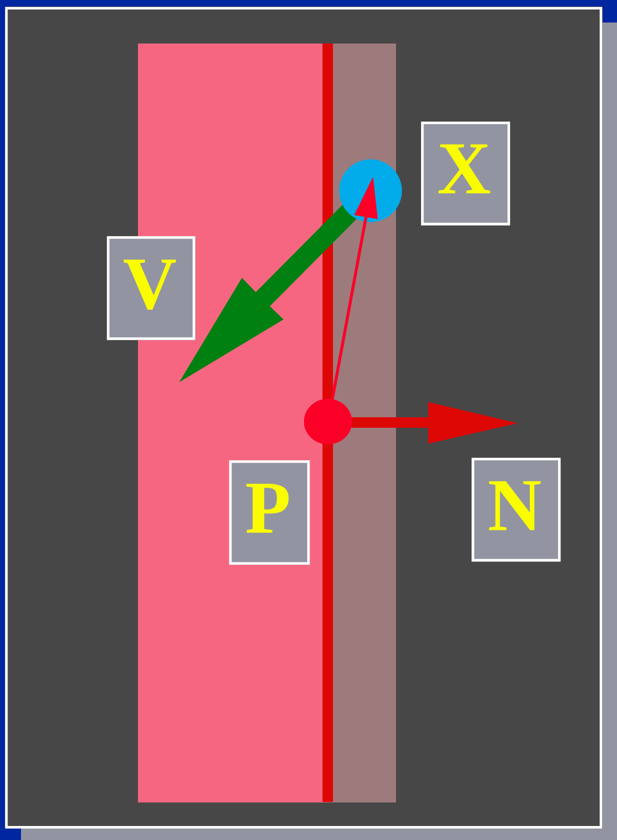
Normal and Tangential Components





$$\mathbf{V}_N = (\mathbf{N}\cdot\mathbf{V})\mathbf{N}$$
$$\mathbf{V}_T = \mathbf{V} - \mathbf{V}_N$$

Collision Detection

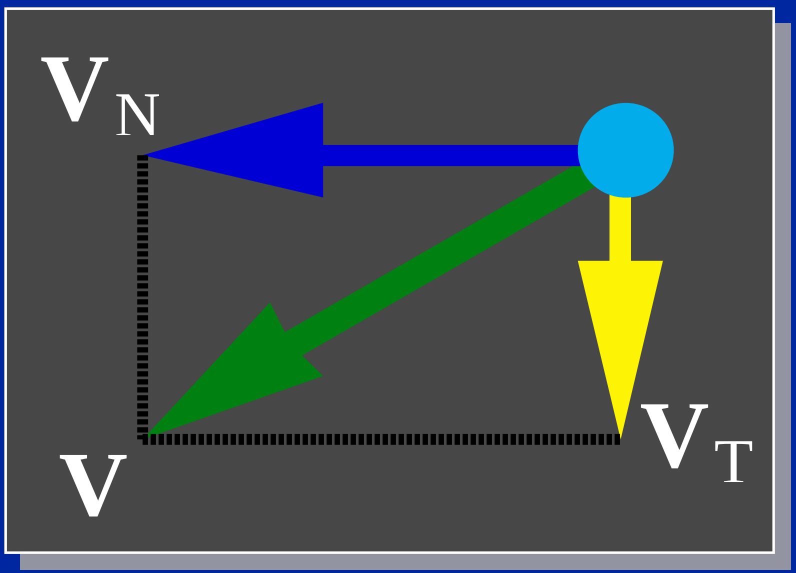


$$(\mathbf{X} - \mathbf{P}) \cdot \mathbf{N} < \varepsilon$$

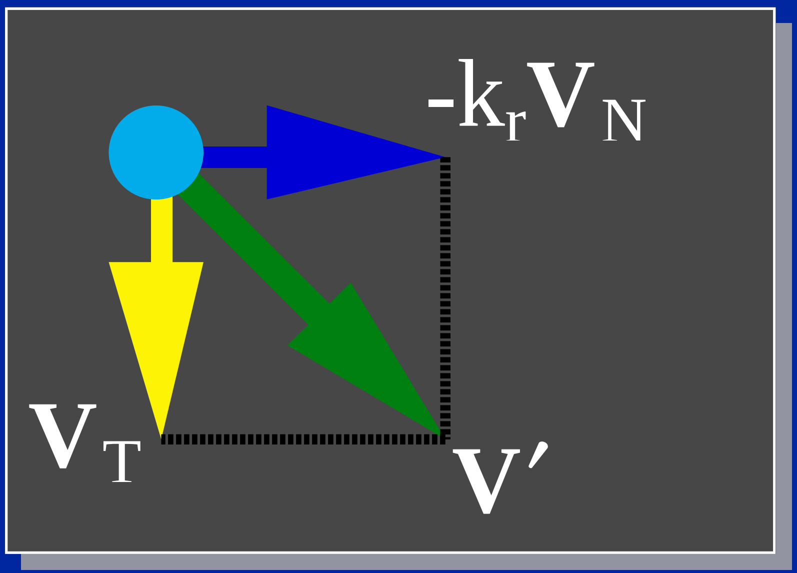
$$\mathbf{N} \cdot \mathbf{V} < 0$$

- Within ε of the wall.
- Heading in.

Collision Response



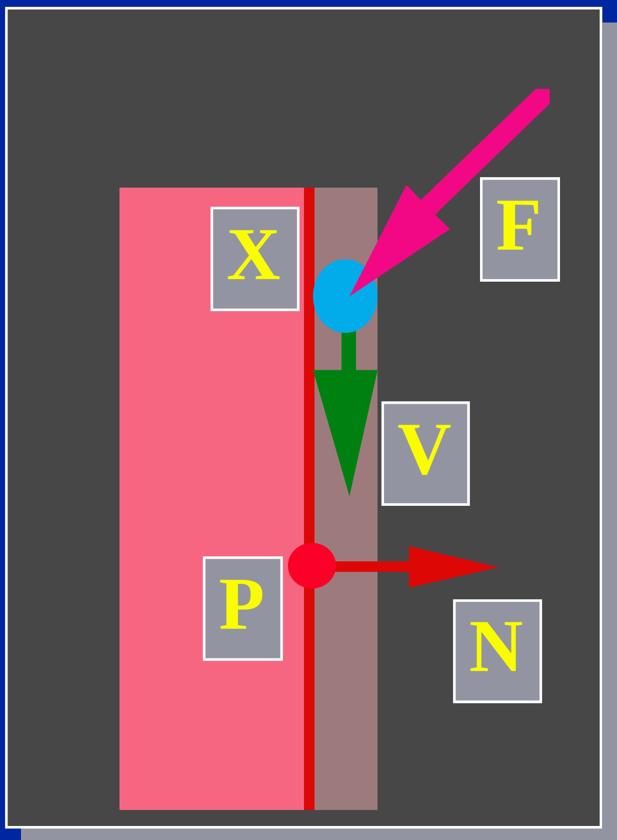
Before



After

$$\mathbf{V}' = \mathbf{V}_T - k_r\mathbf{V}_N$$

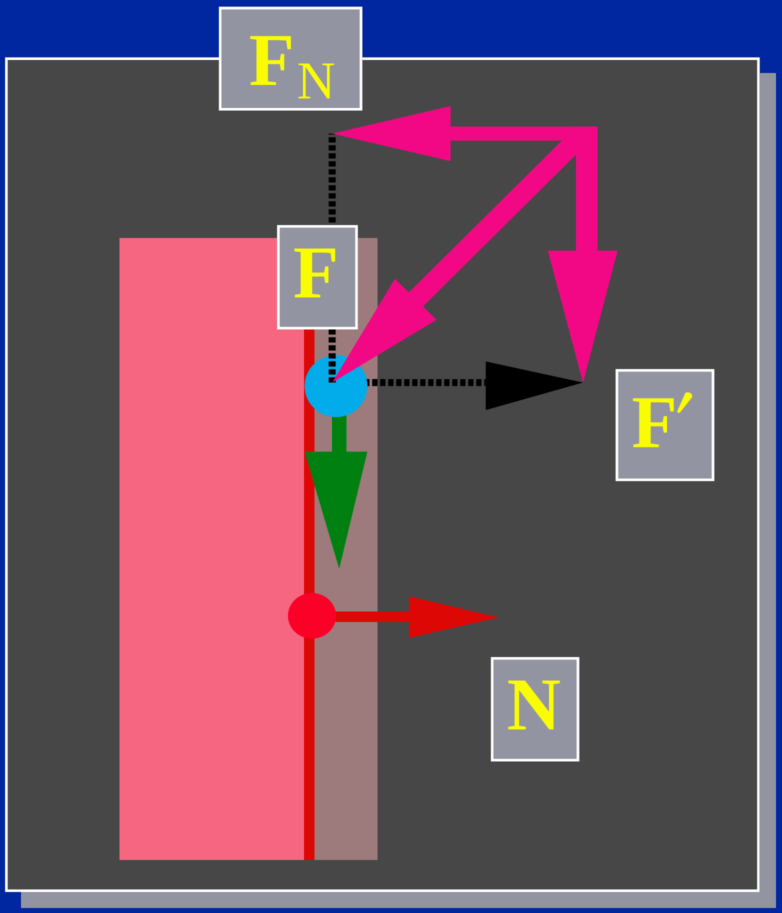
Conditions for Contact



$$|(\mathbf{X} - \mathbf{P}) \cdot \mathbf{N}| < \varepsilon$$

$$|\mathbf{N} \cdot \mathbf{V}| < \varepsilon$$

- On the wall
- Moving along the wall
- Pushing against the wall



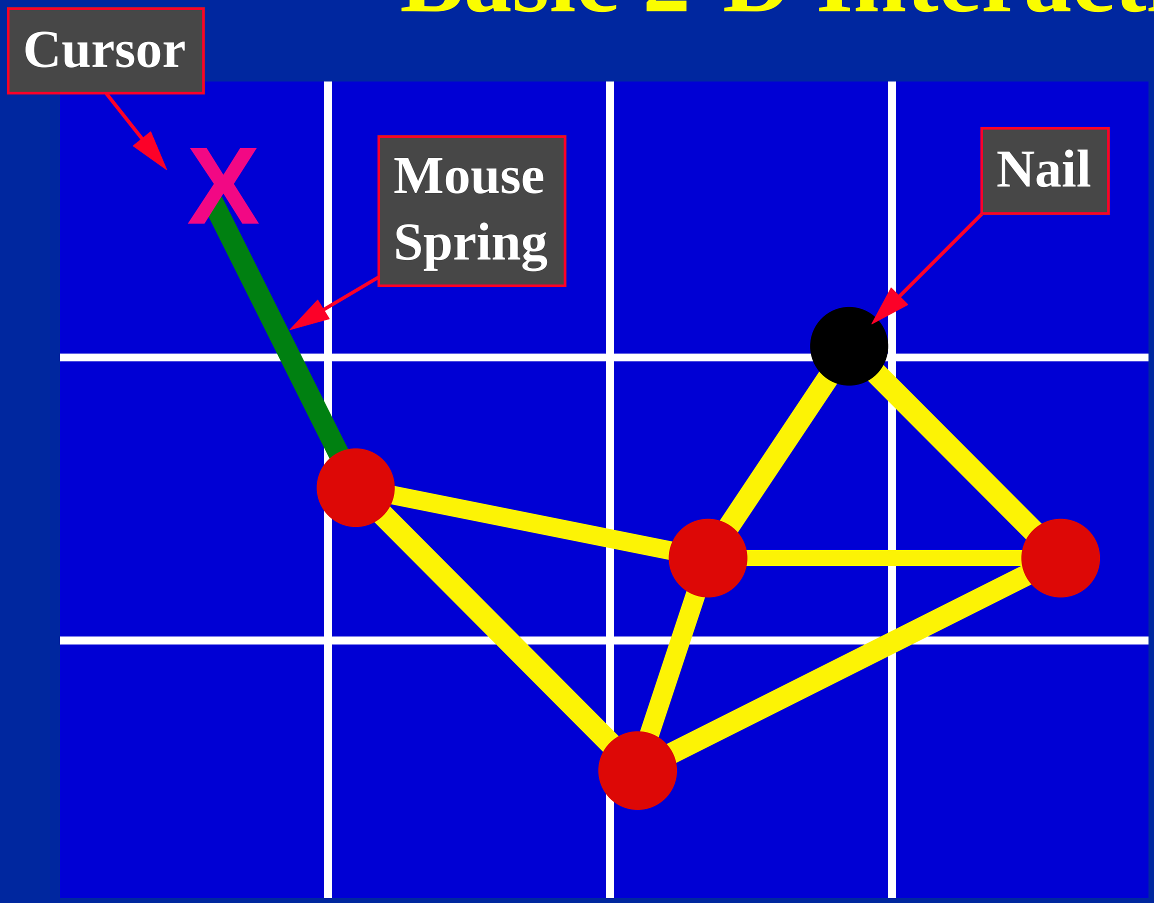
Contact Force

$$F' = F_T$$

The wall pushes back, cancelling the normal component of F.

(An example of a *constraint force.*)

Basic 2-D Interaction



Operations:

- Create
- Attach
- Drag
- Nail

Try this at home!

The notes give you everything you need to build a basic interactive mass/spring simulator—try it.