Artificial Intelligence

CS 165A

Oct 22, 2020

Instructor: Prof. Yu-Xiang Wang

Today

→ Problem Solving by Search

→ Search algorithms

Student Feedback (from past two weeks)

- “Your handwriting is sometimes not legible. ”
  - Will do.

- “Sharing annotated lecture slides”
  - Will do.

- “computer engineering peers and I are struggling a lot with the gradient/multivariable concepts”
  - Don’t be scared. The concept of gradient is natural. Try visualize in 2D, 3D and work with examples, such as $f(x,y) = x^2 + xy$ to build intuition.

- “I feel that I am not doing well in HW1. Am I gonna fail?”
  - Probably no. Evaluation is quite lenient this quarter (late days, partial credits). Everyone can keep up.

Recap of the last lecture

- Three steps in modelling with Bayesian networks
- Inference with Bayesian networks using only CPTs
- Three equivalent ways of describing structures of a joint distribution
  - Factorization ⇔ DAG ⇔ the set of conditional independences
- Prove conditional independence by definition.

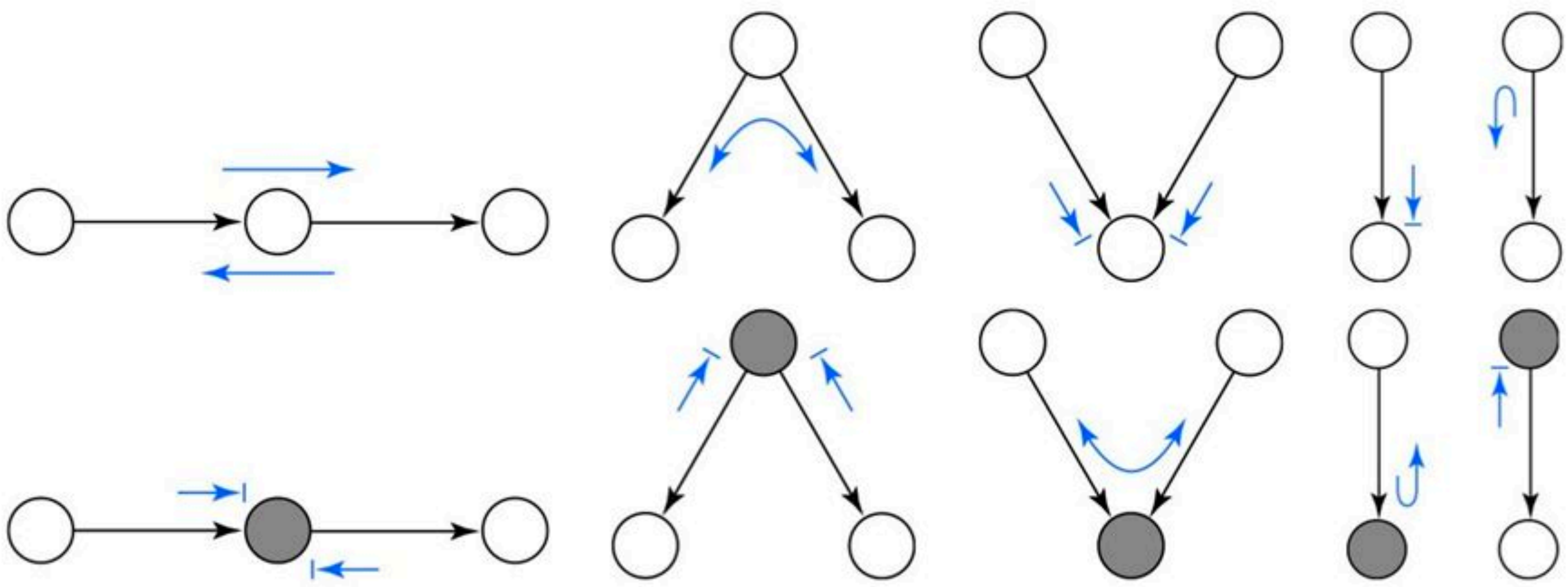
Recap of the last lecture

- Reading conditional independences from the DAG itself.

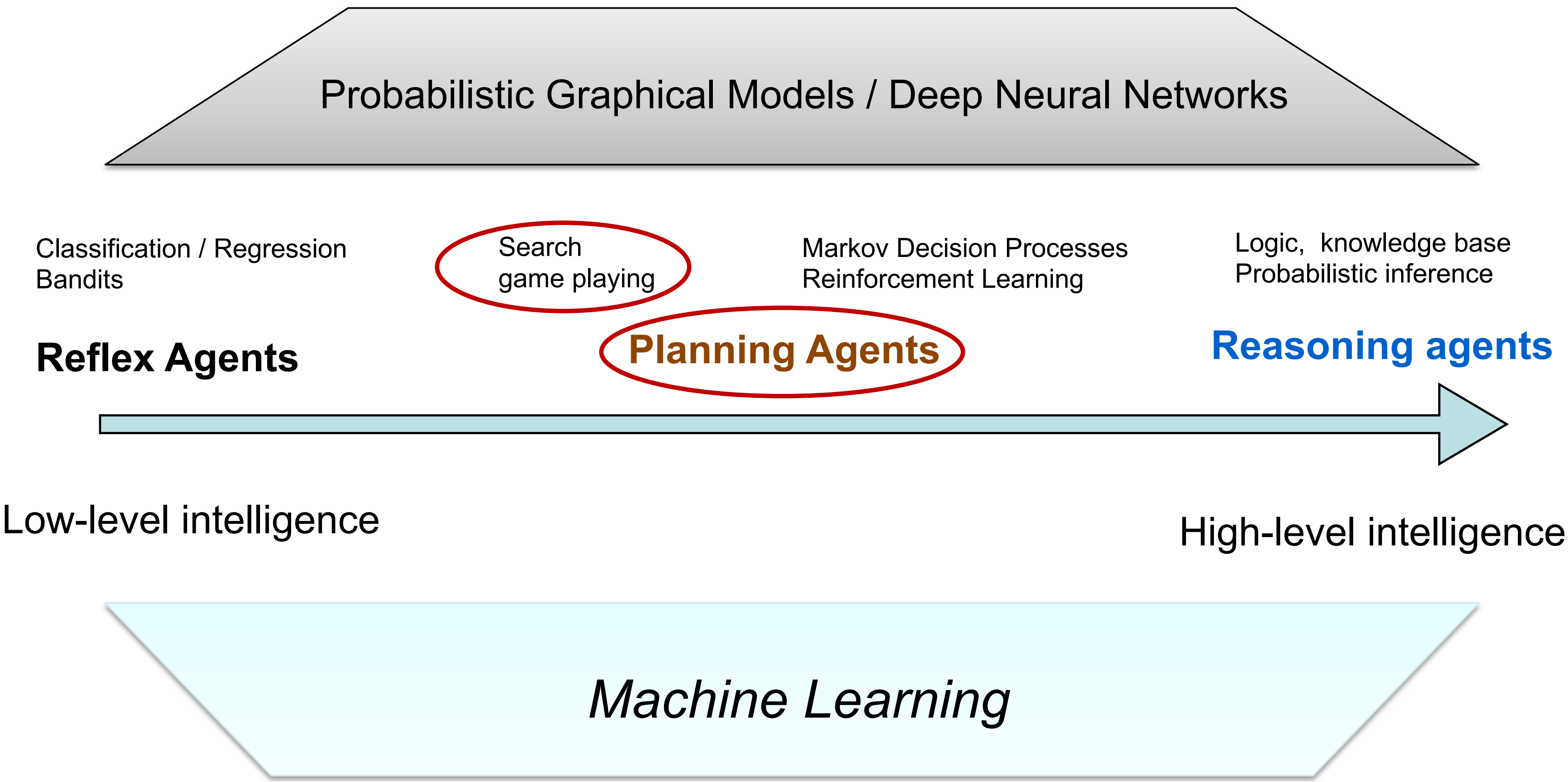
- d-separation
  - Three canonical graphs

- Bayes ball algorithm for determining whether $\mathbf{X} \perp \mathbf{Z} \mid \mathbf{Y}$
  - Bounce the ball from any node in X by following the ten rules
  - If any ball reaches any node in Z, then return "False"
  - Otherwise, return "True"

The Ten Rules of Bayes Ball Algorithm

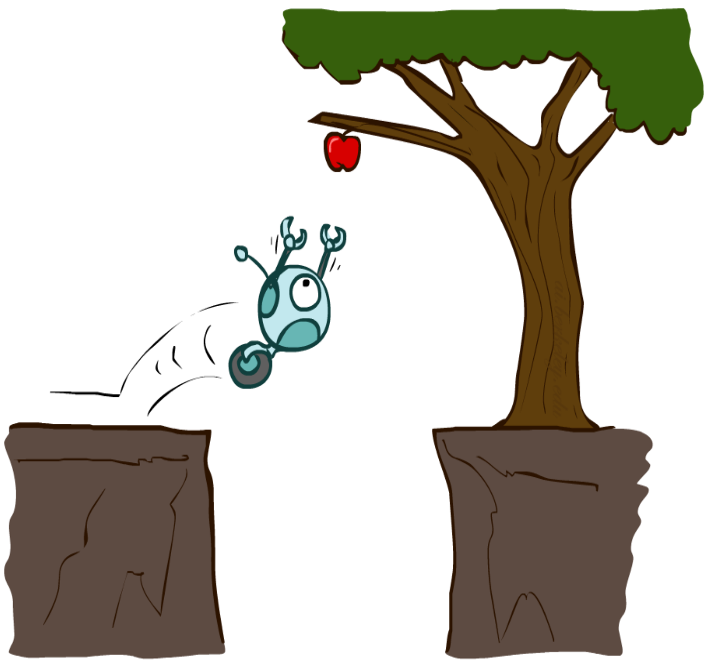


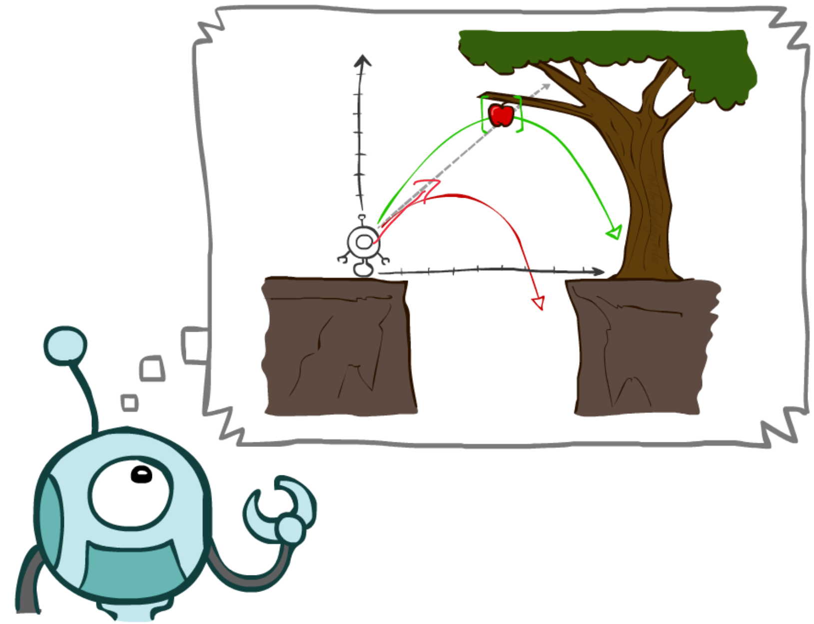
Structure of the course



(Again this idea is adapted from Percy Liang's teachings)

Reflex Agents vs. Planning agent





(illustration credit: Dan Klein)

- Reflex agents act based on immediate observation / memory; often optimizes immediate reward.
- Planning agent looks further into the future and "try out" different sequences of actions --- in its mind --- before taking an action; optimizes long-term reward.

Modeling-Learning-Inference Paradigm

	Modeling	Learning	Inference
Classifier agent (Spam filter)	Feature engineering Hypothesis class	Minimize Error rate	trivial
Probabilistic Inference agent (Sherlock)	Joint distribution Draw edges in BN Conditional independences	Fitting the CPTs with MLE	Marginalization (conceptually easy)
Search agents	State-Space-diagram	Environment given (learn edge weights)	**Nontrivial search algorithms**

Search sequence of lectures

- Today: Finish Graphical models. Start "Search"
- Oct 27: Search algorithms
- Oct 29: Minimax search and game playing
- Nov 3: Finish "search" + Midterm review. HW2 Due.

- Recommended readings on search:
  - AIMA Ch 3, Ch 5.1-5.3.

Remaining time today

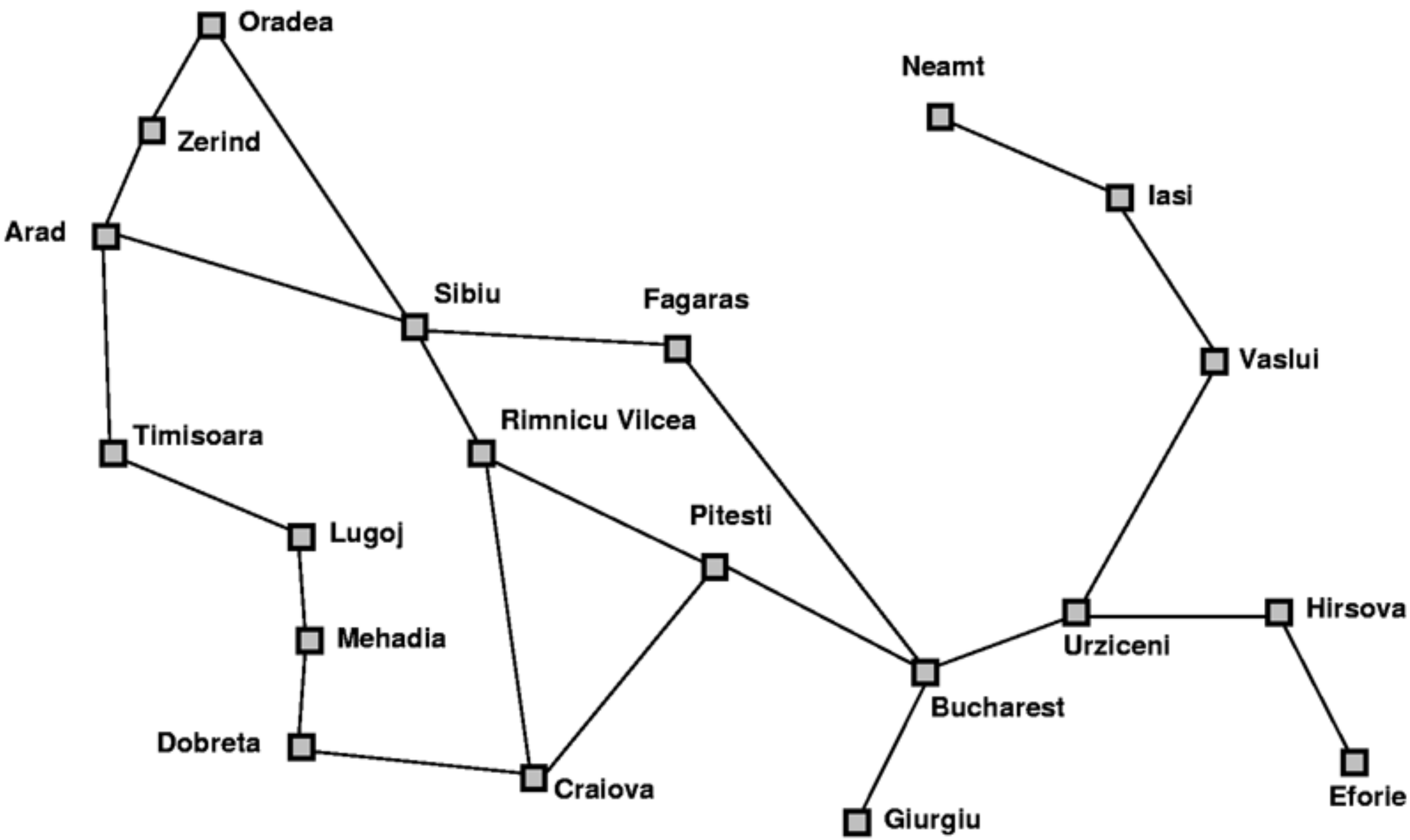
- Formulating problems as search problems
- Basic algorithms for search

Example: Romania

You're in Arad, Romania, and you need to get to Bucharest as quickly as possible to catch your flight.

- Formulate problem
  - States: Various cities
  - Operators: Drive between cities
- Formulate goal
  - Be in Bucharest before flight leaves
- Find solution
  - Actual sequence of cities from Arad to Bucharest
  - Minimize driving distance/time

Romania (cont.)



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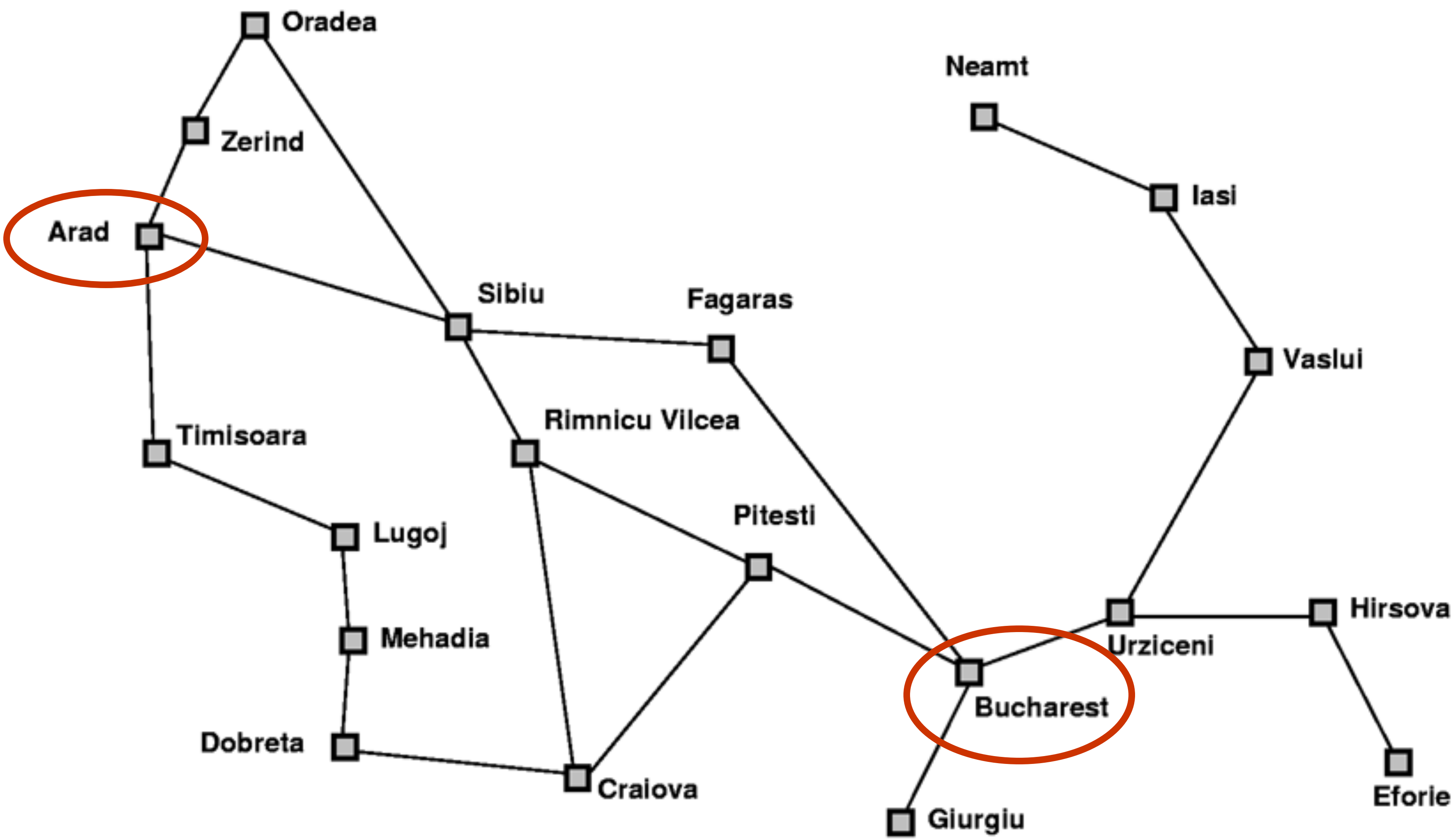
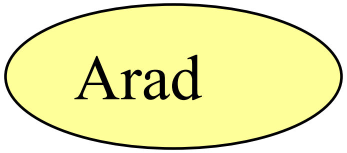


Romania (cont.)

Problem description **<{S}, S_0, {S_G}, {O}, {g}>**

- **{S}** – cities (c_i)
- $\mathbf{S_0}$ – Arad
- $\mathbf{S_G}$ – Bucharest
  - G(S) – Is the current state (S) Bucharest?
- {O}: { $c_i \rightarrow c_j$, for some *i* and *j* }
- $\mathbf{g_{ij}}$
  - Driving distance between c_i and c_j?
  - Time to drive from c_i to c_j?
  - 1?

Possible paths



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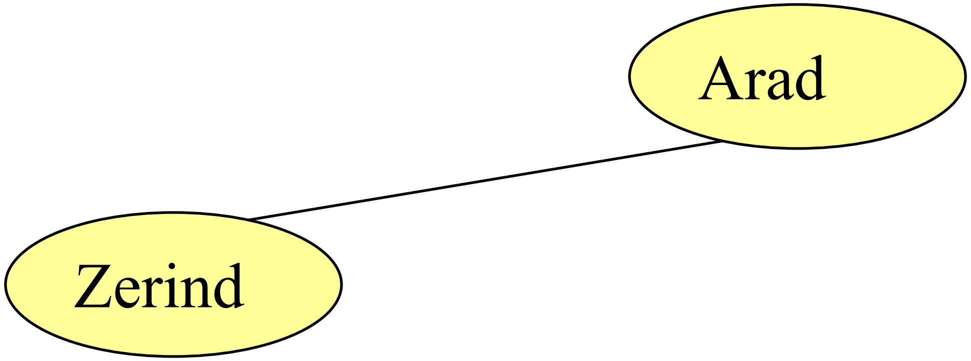

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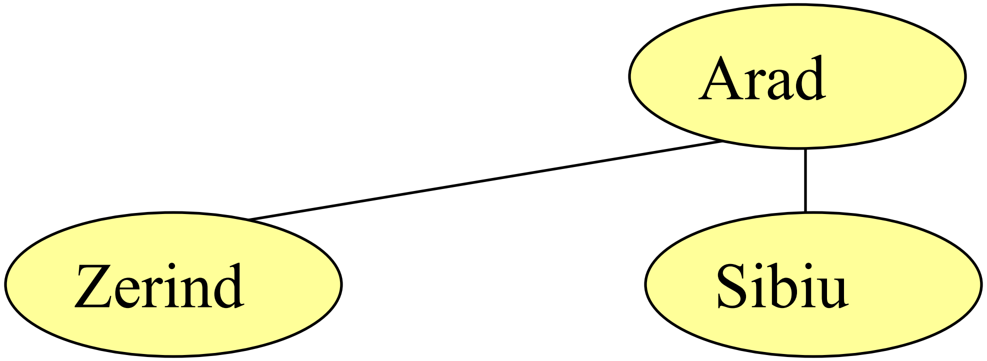

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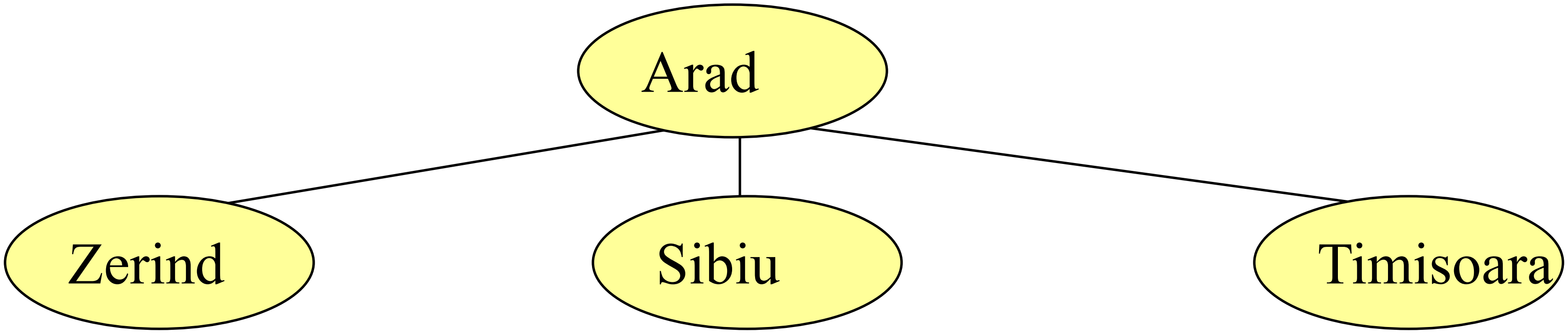

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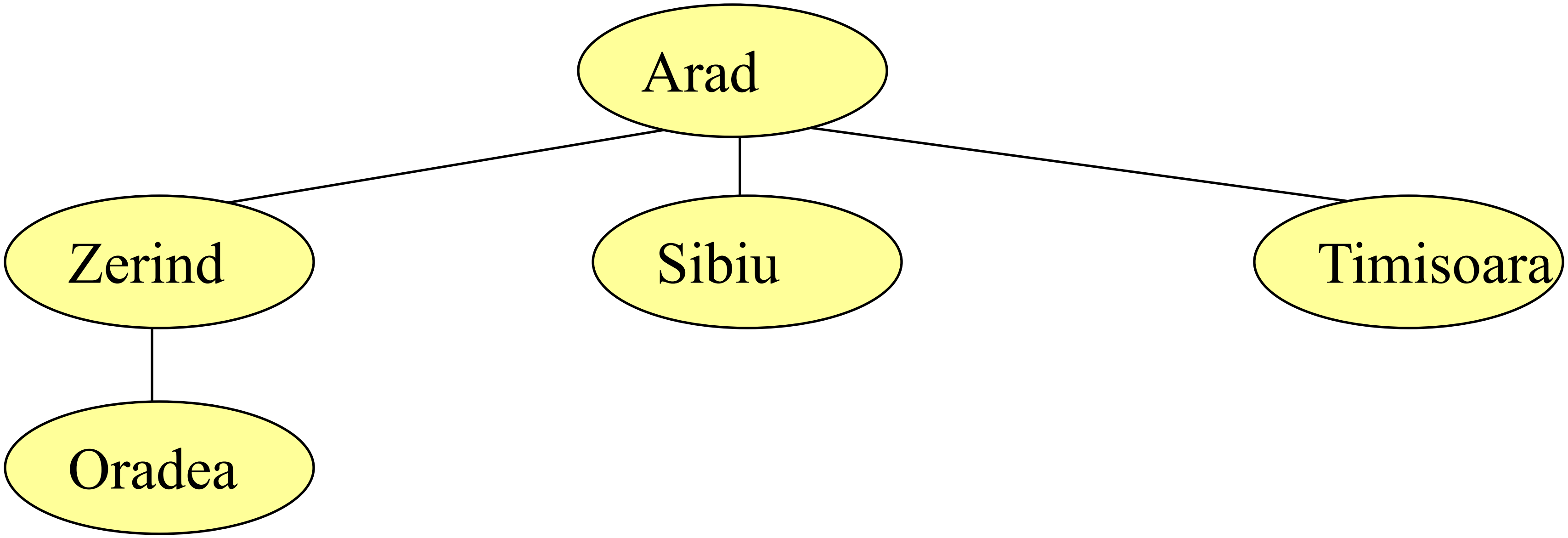

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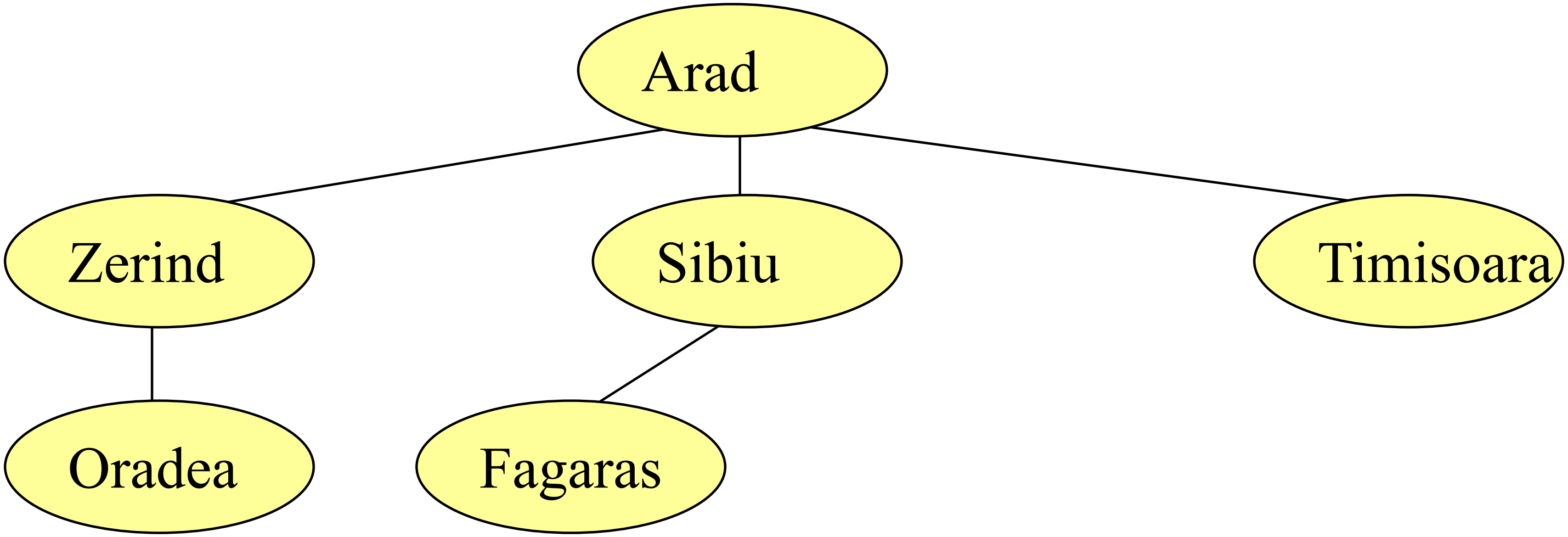

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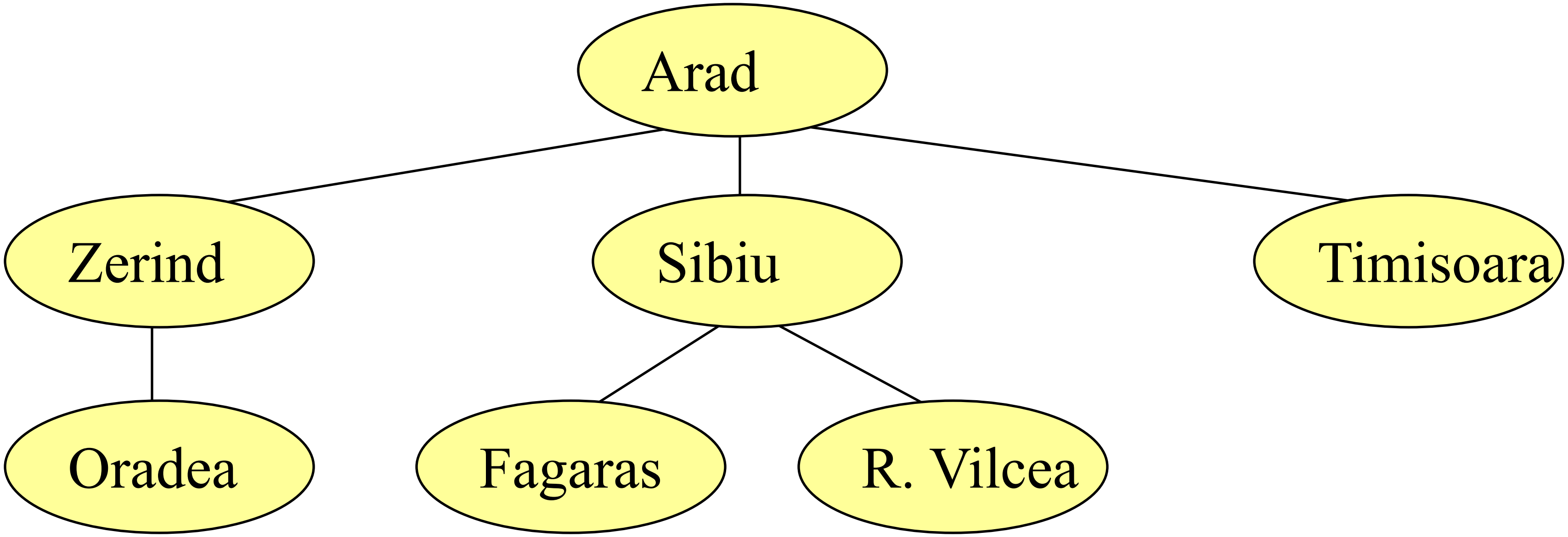

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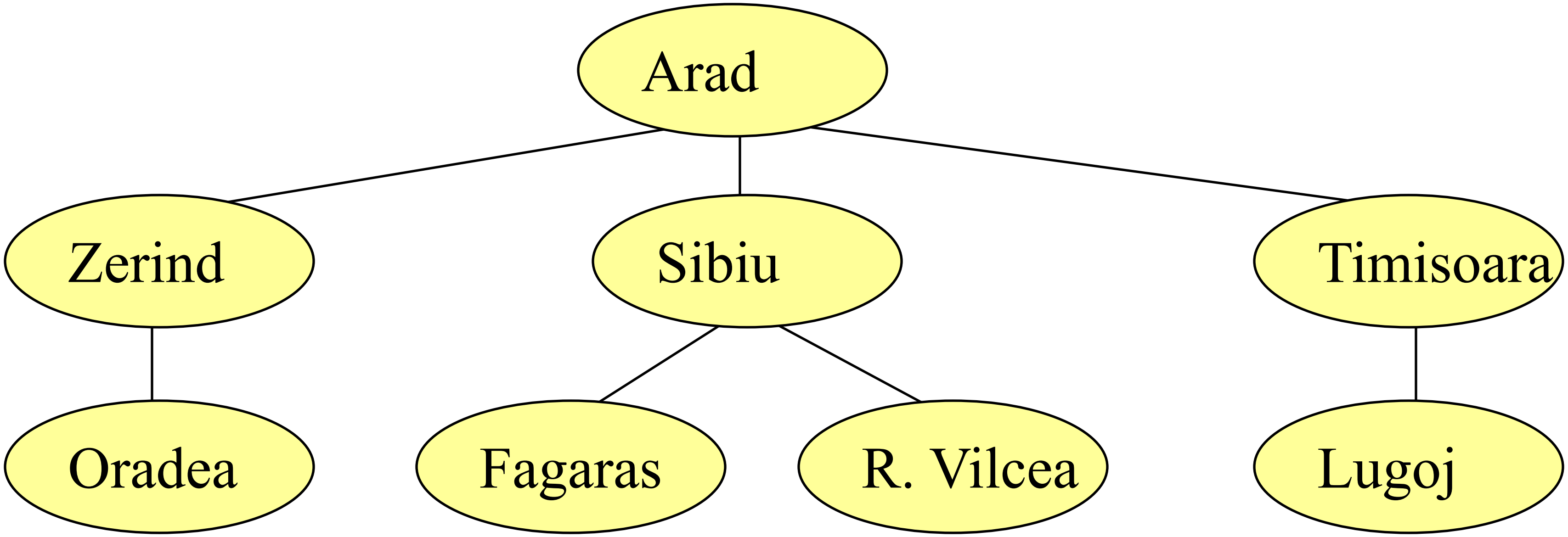

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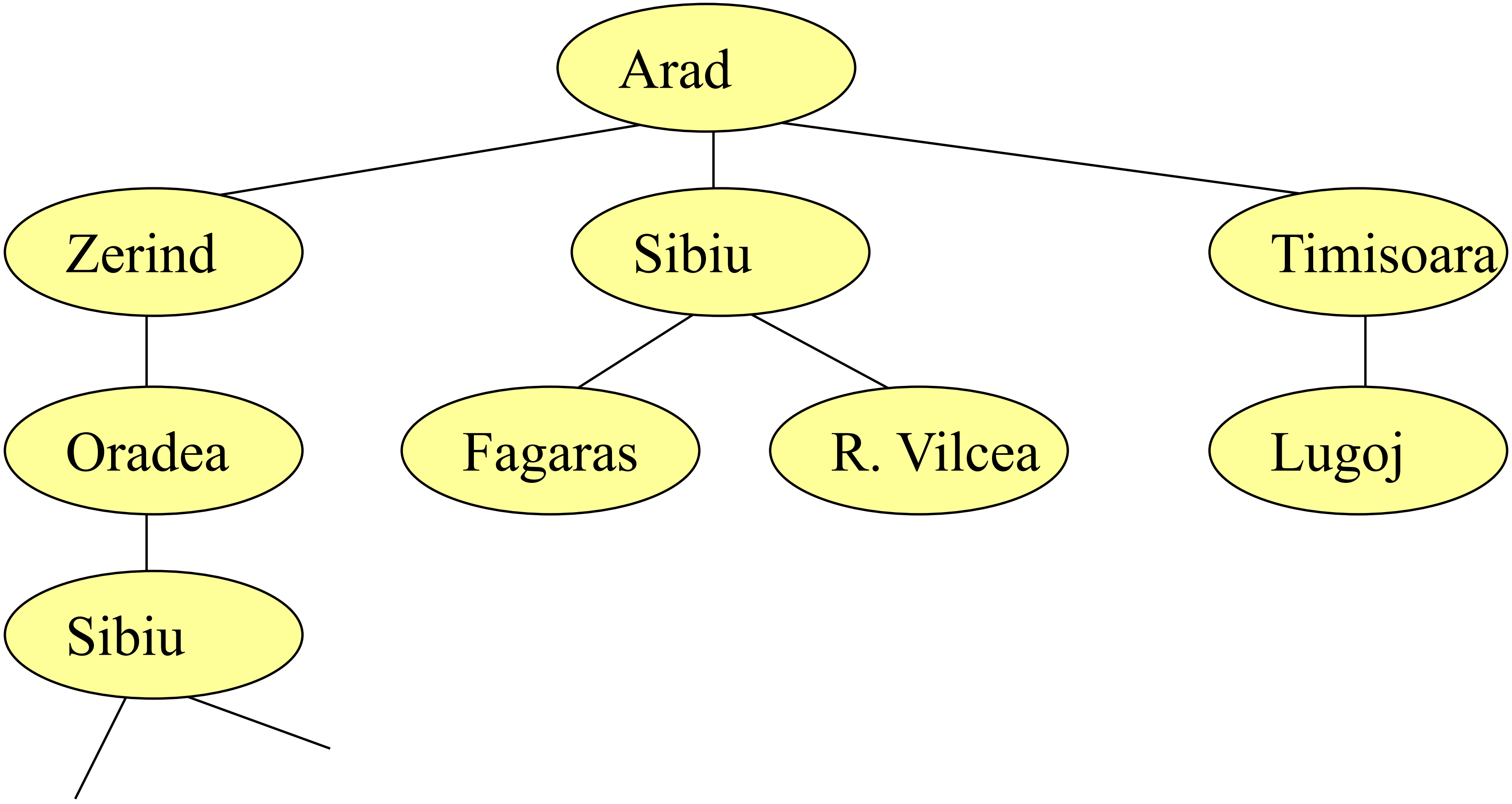

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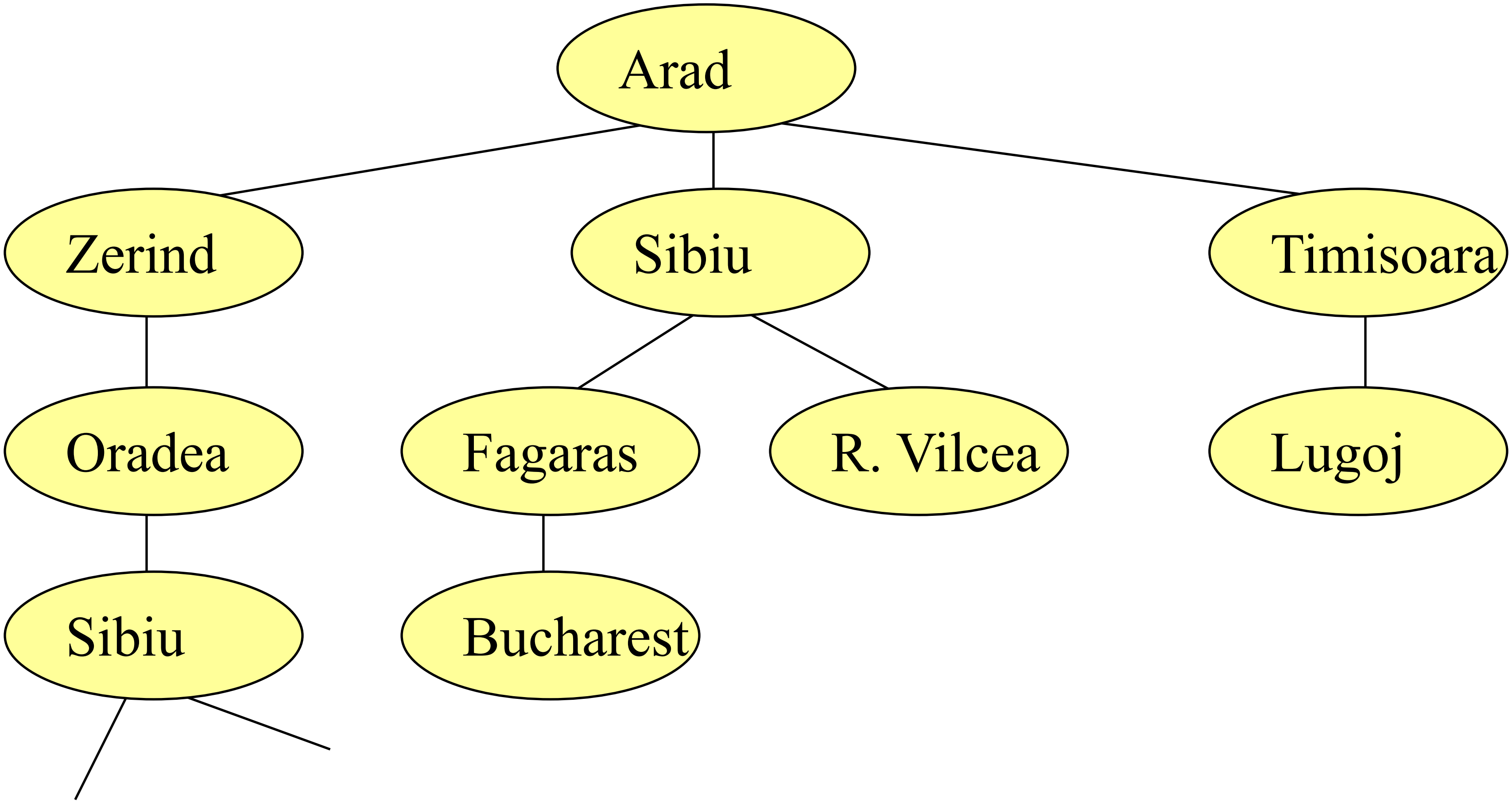

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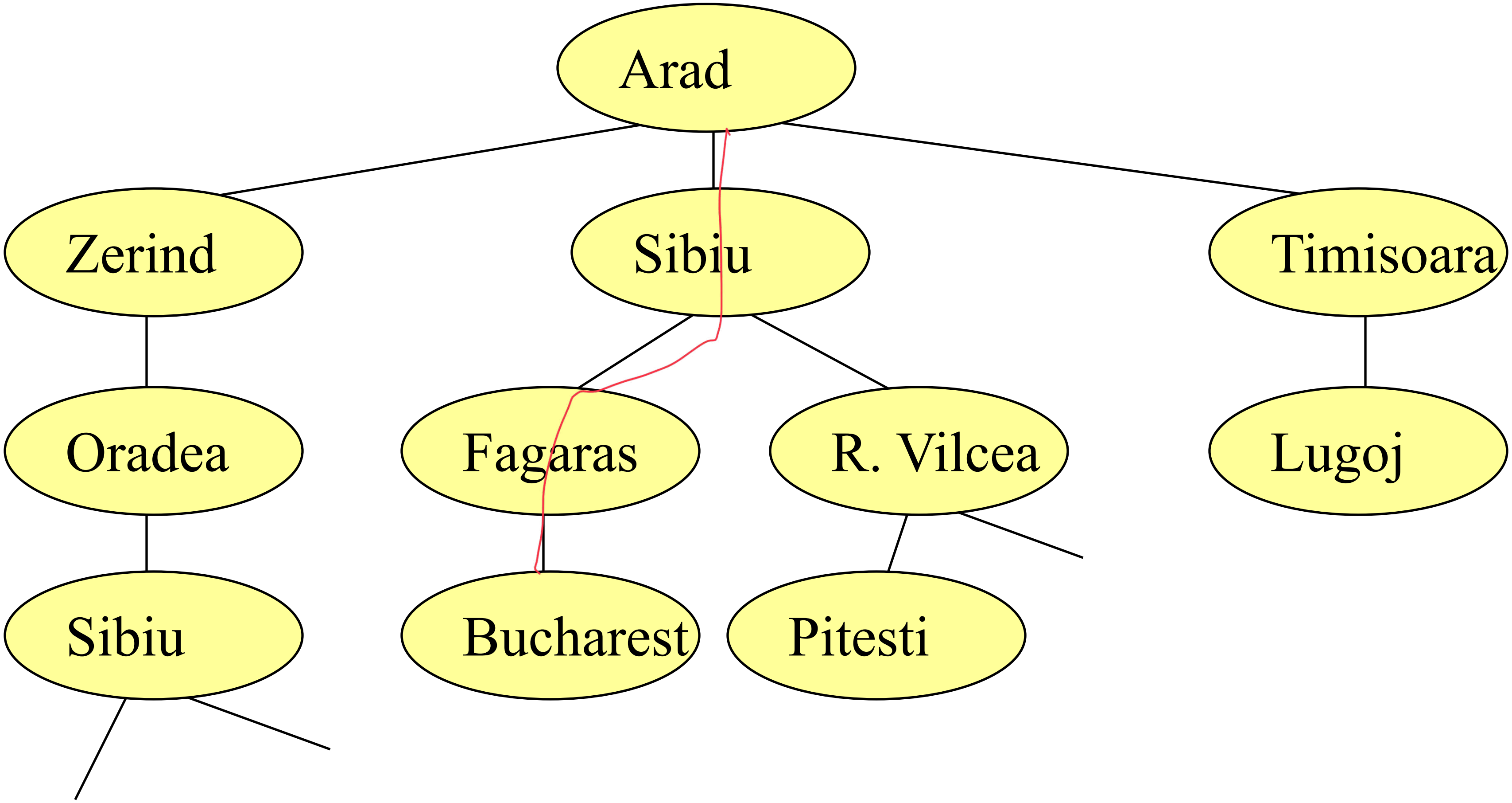

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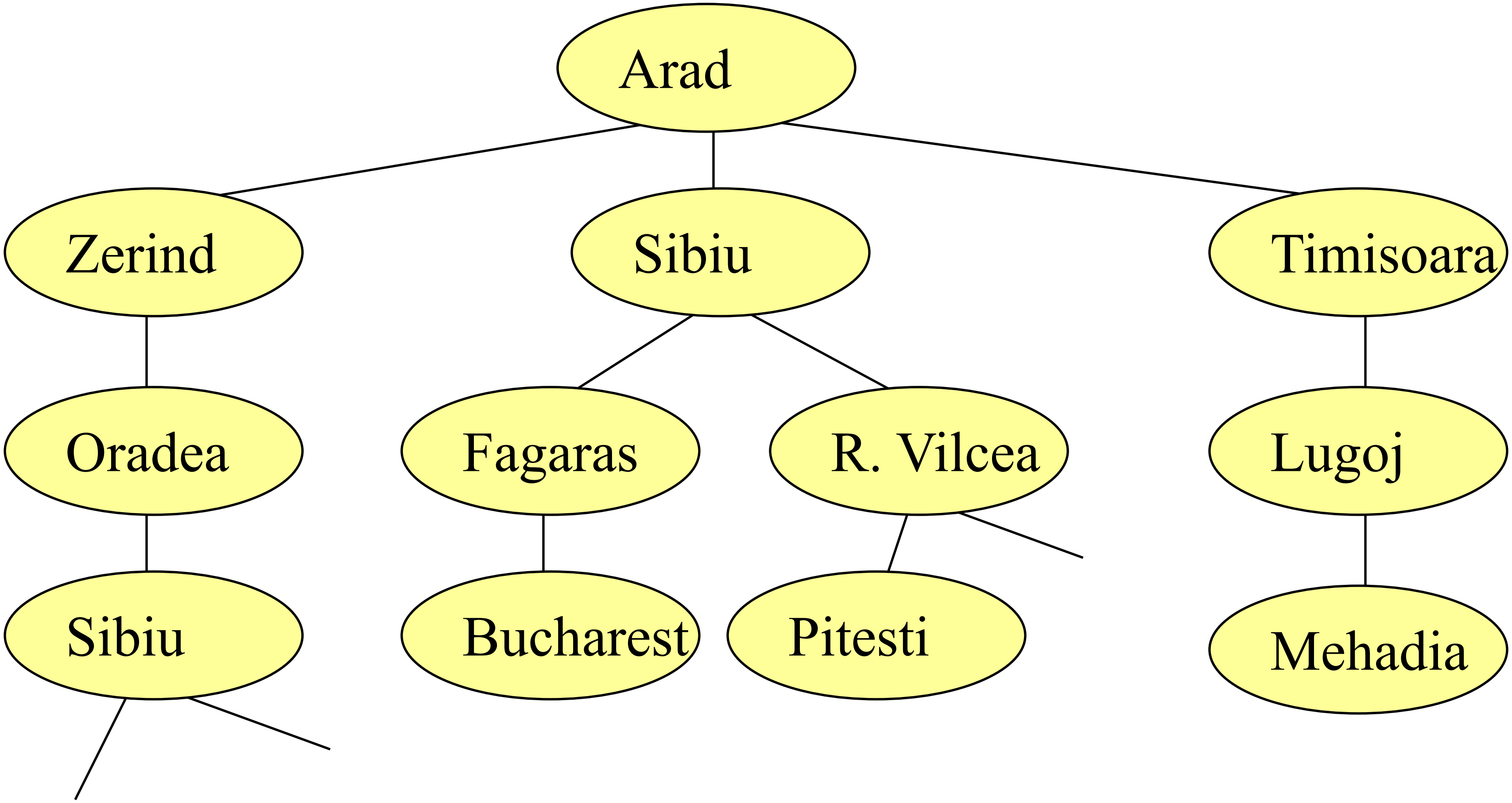

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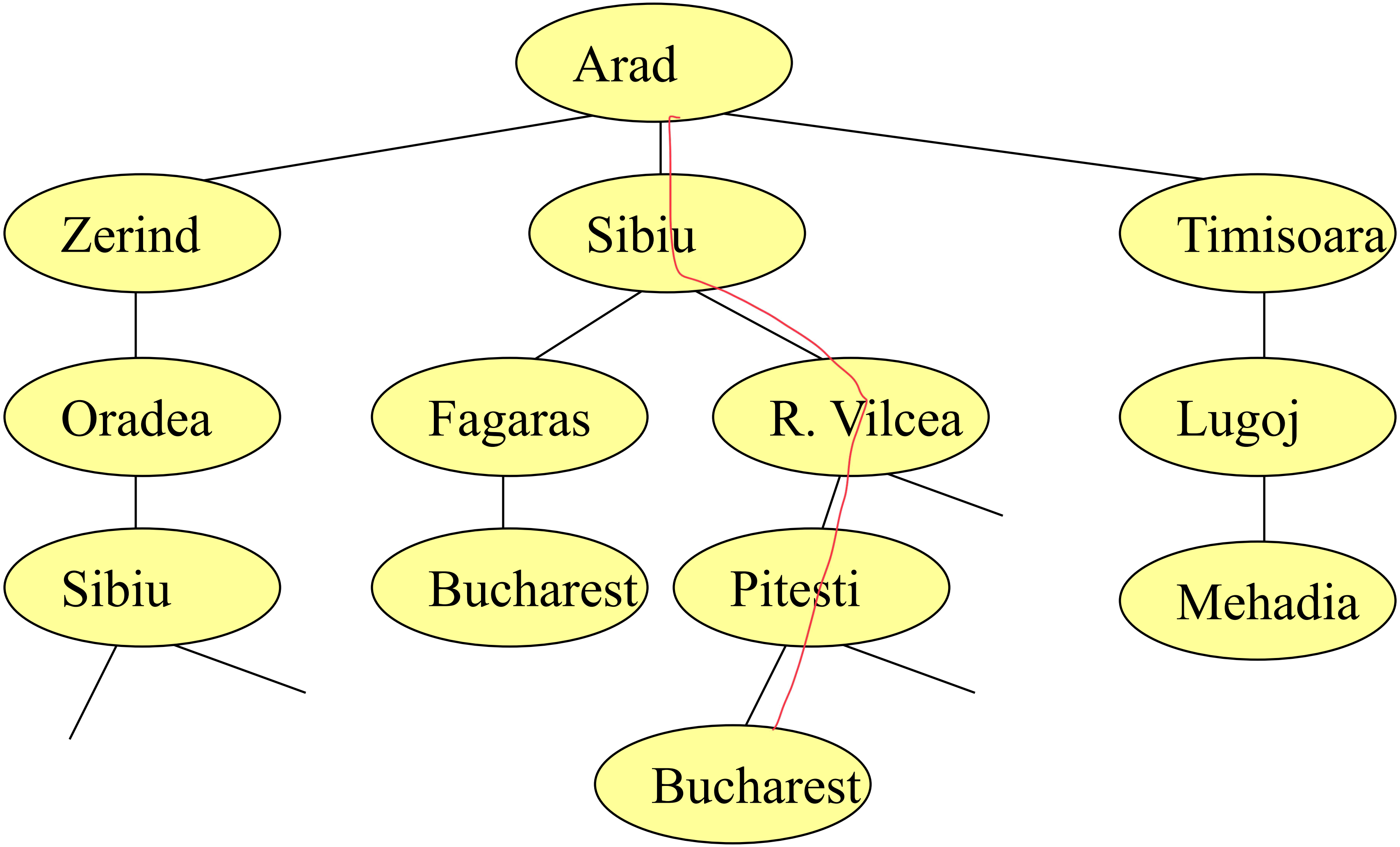

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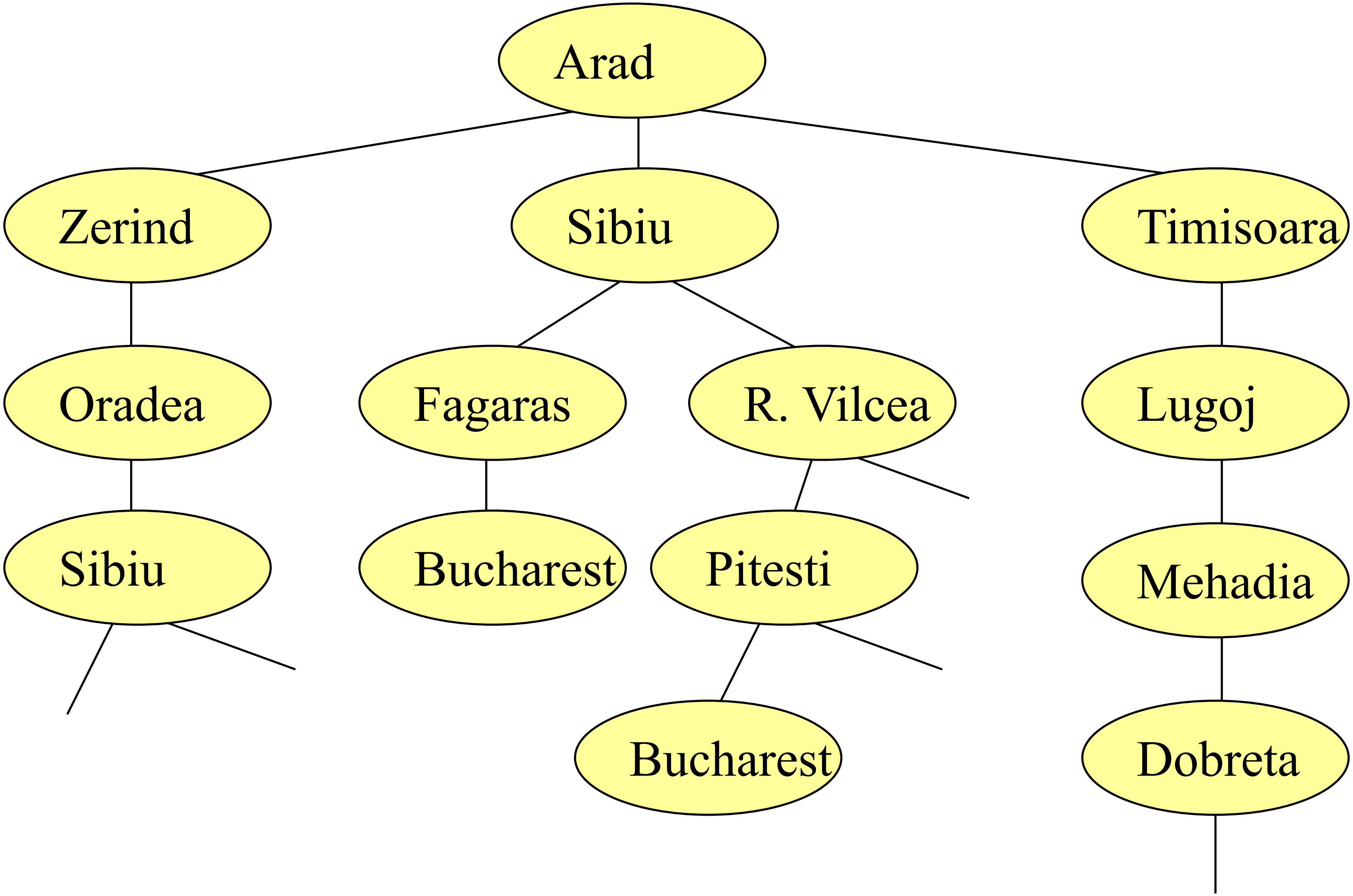

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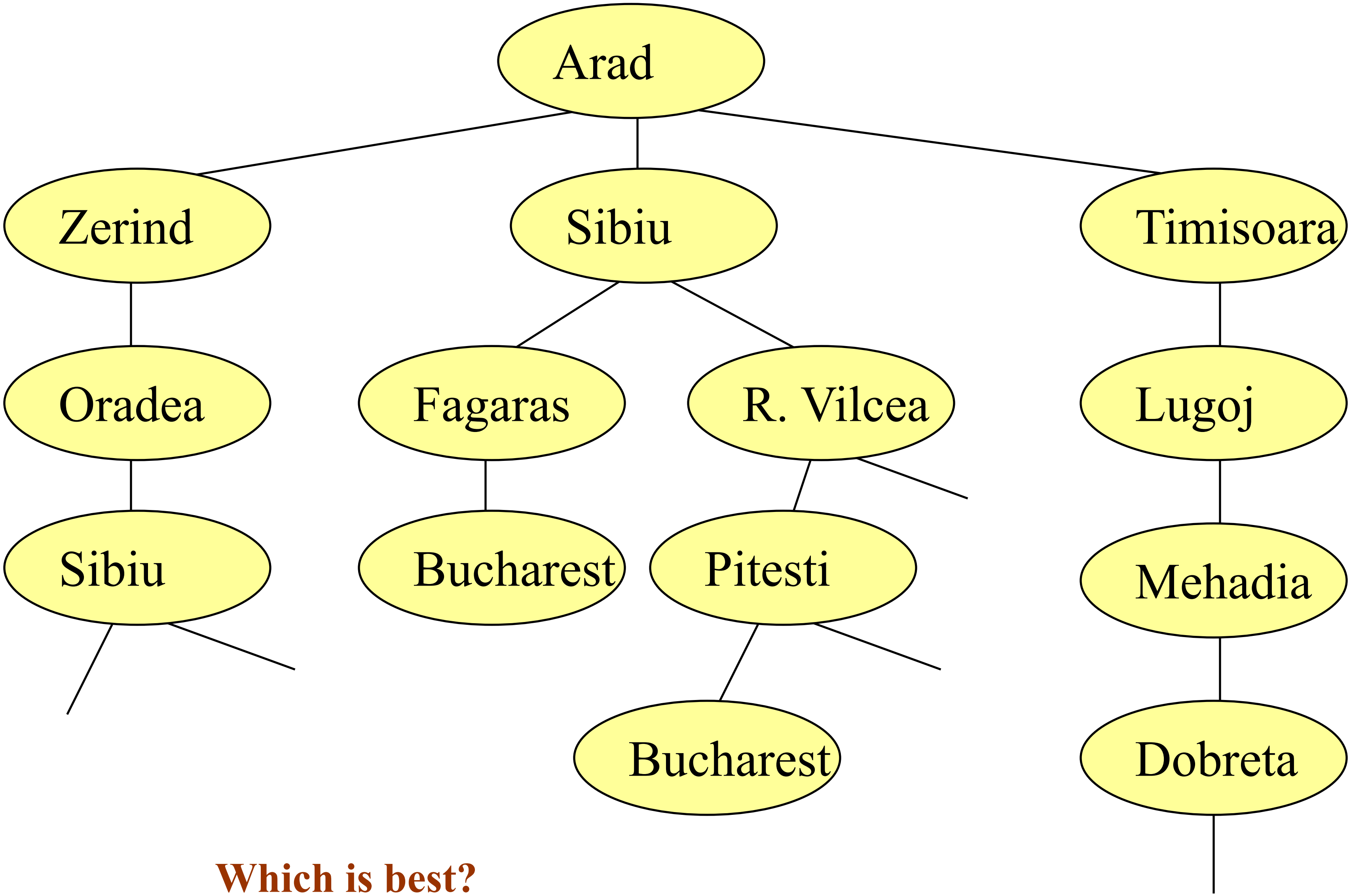
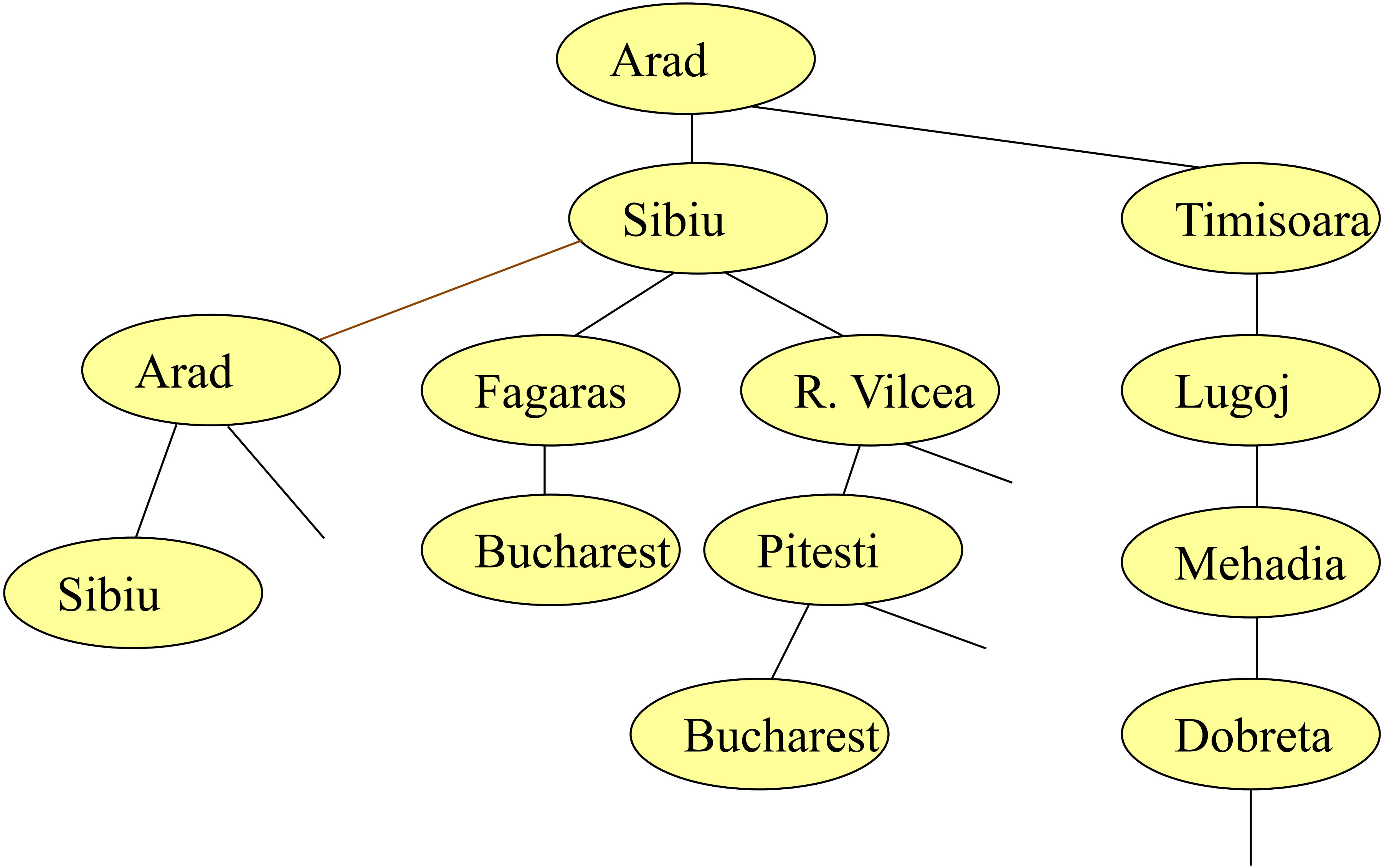


Which is best?

Should we consider cycles?



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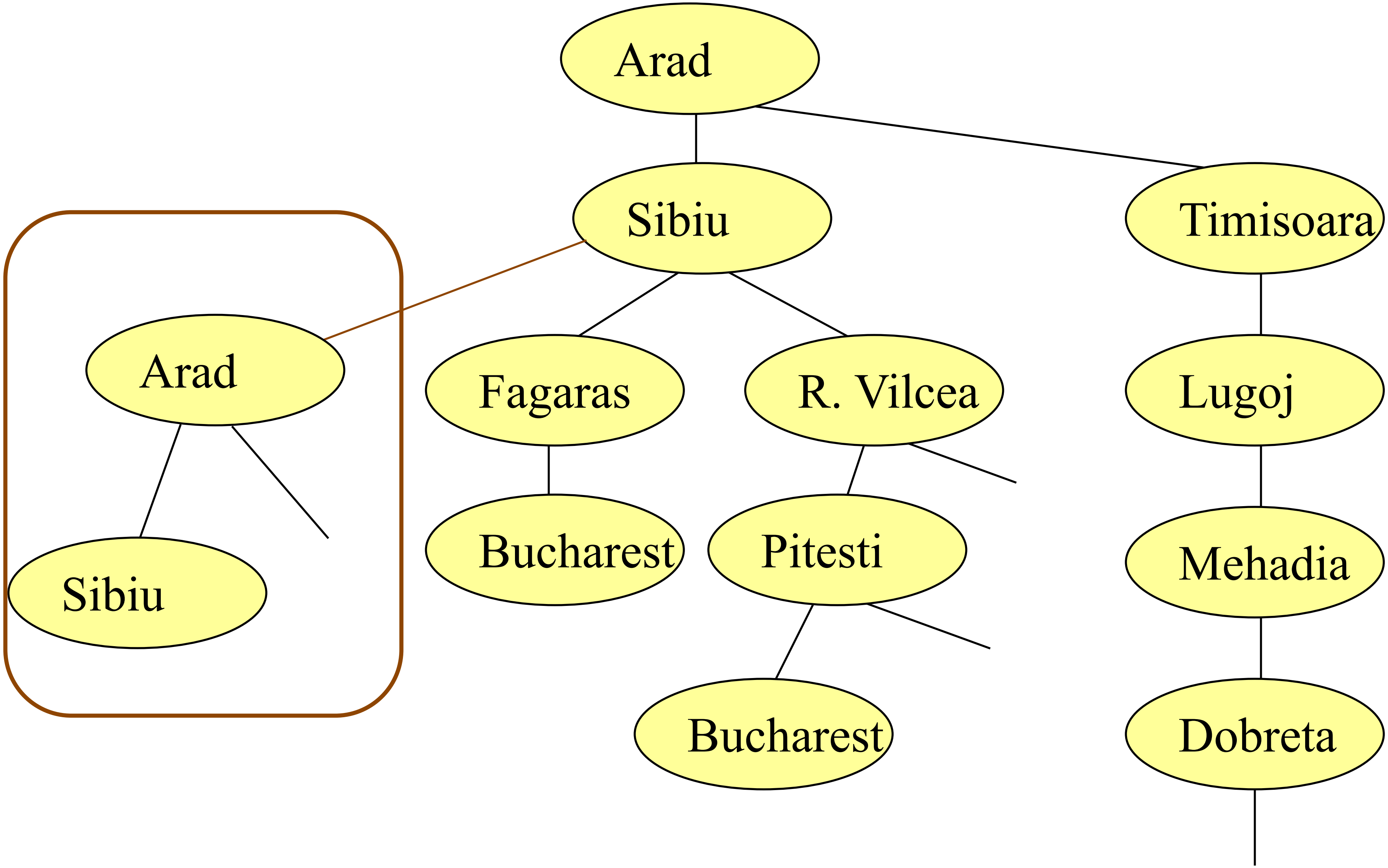

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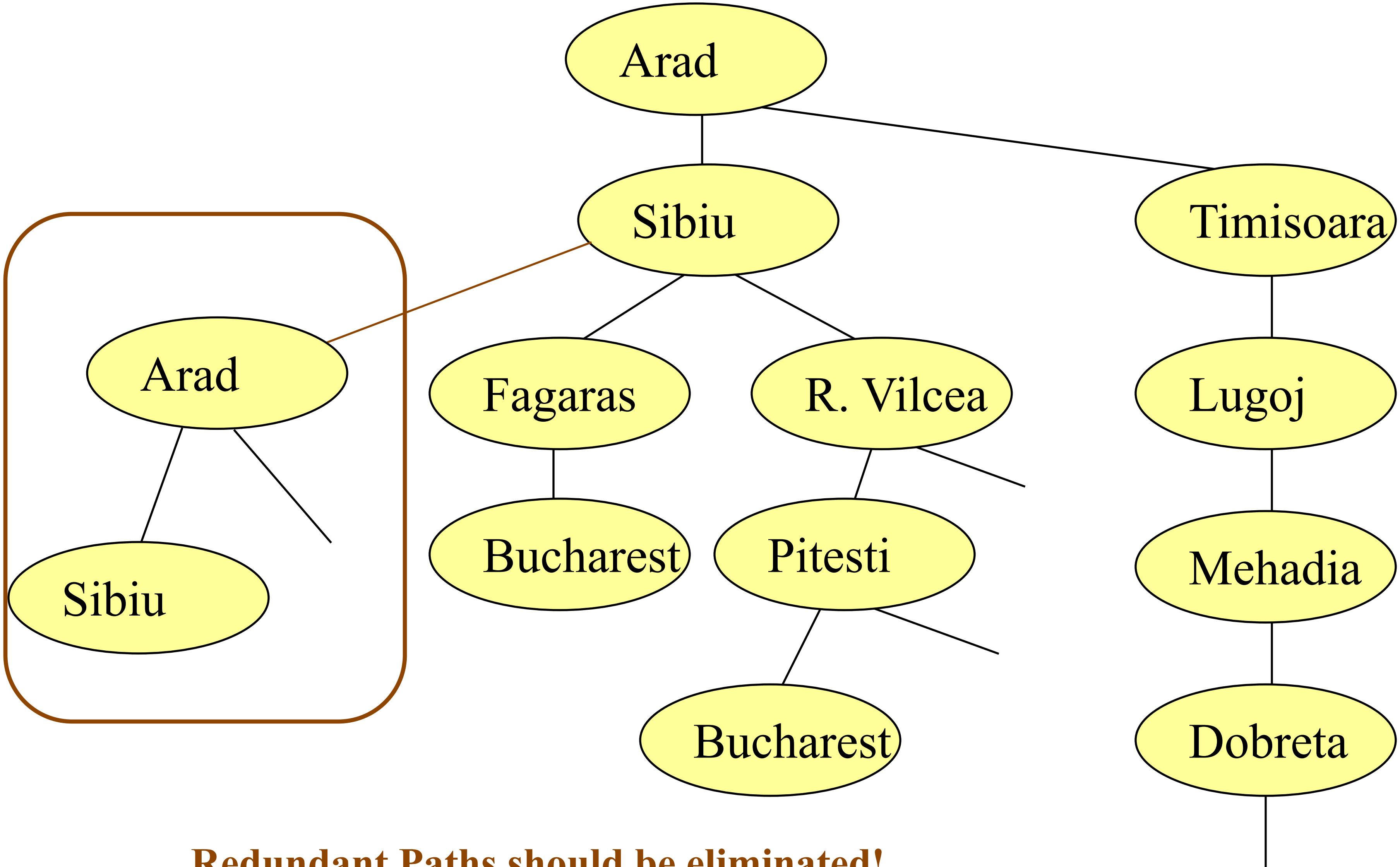

Redundant Paths should be eliminated!

Branching Factor and Depth

- If there are *b* possible choices at each state, then the **branching factor** is *b*

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- Ouch…. Combinatorial explosion!

Abstraction

- The real world is highly complex!
  - The state space must be *abstracted* for problem-solving
    - Simplify and aggregate
      - Can't represent all the details
- Choosing a good abstraction
  - Keep only those relevant for the problem
  - Remove as much detail as possible *while retaining validity*

Problem Solving Agents

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- Formulate, search, execute (action)

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  - Depends on the goal, the path cost function, the future sequence of actions….
- Agent's strategy: Formulate, Search, and Execute
  - This is *offline* problem solving

State-Space Diagrams

- State-space description can be represented by a state-space diagram, which shows
  - States (incl. initial and goal)
  - Operators/actions (state transitions)
  - Path costs

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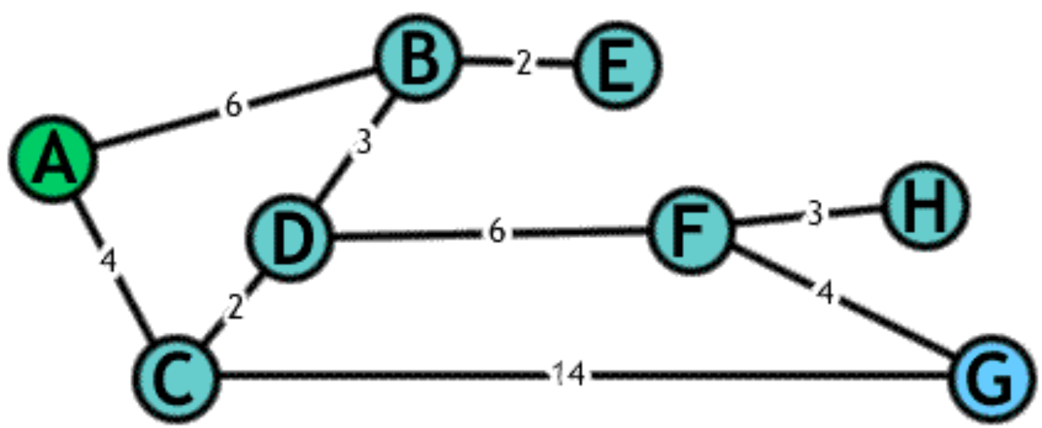
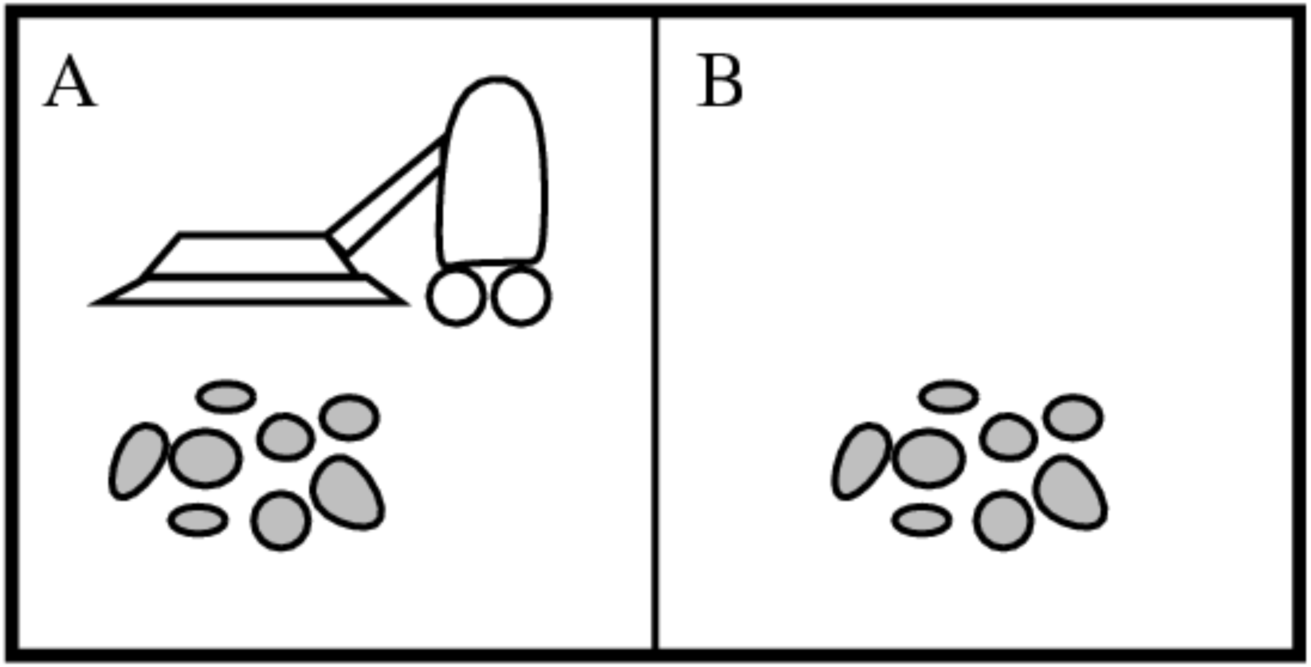


Typical assumptions

- Environment is observable
- Environment is static
- Environment is discrete
- Environment is deterministic

Example: The Vacuum World



The Vacuum World

- Simplified world: 2 grids

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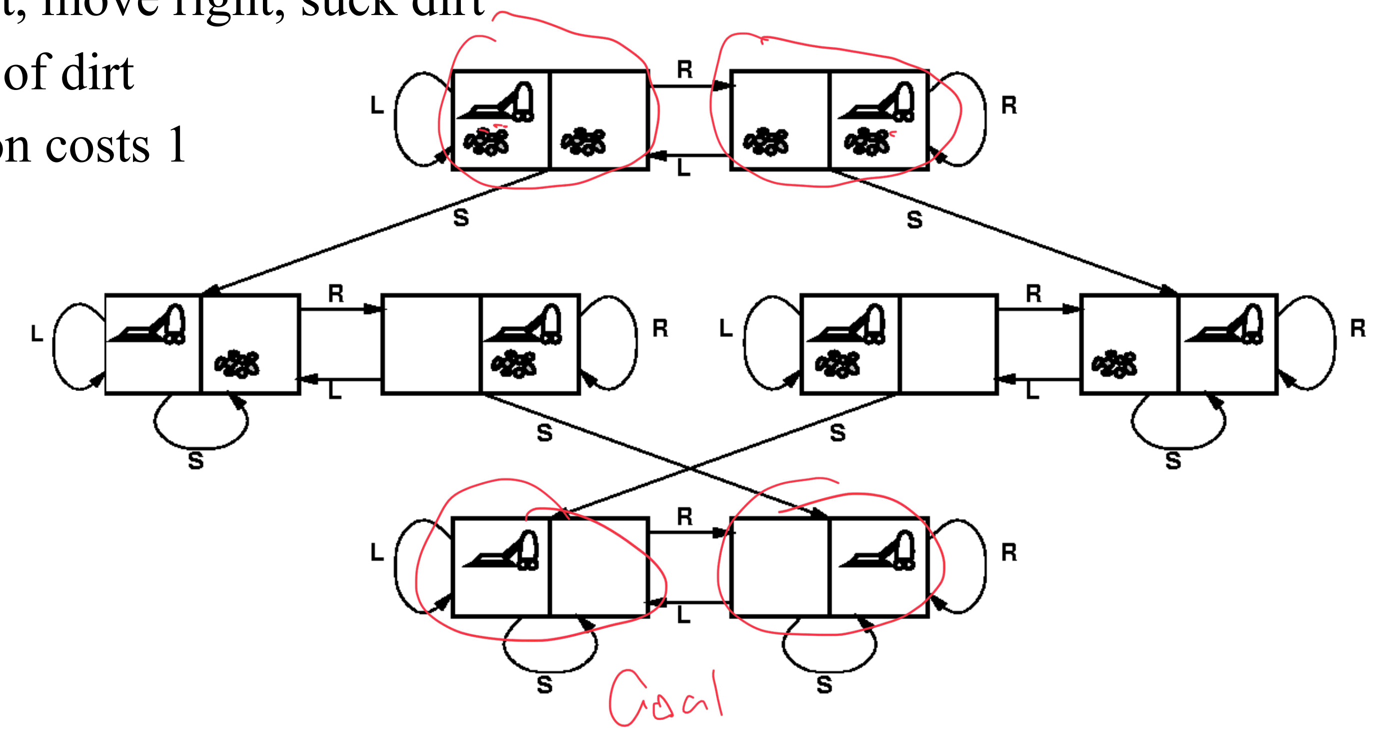
States: Location of vacuum, dirt in grids

Operators: Move left, move right, suck dirt

Goal test: Grids free of dirt

Path cost: Each action costs 1



The Vacuum World

- Simplified world: 2 grids

8

$2^2 \times 2$

1011

1110

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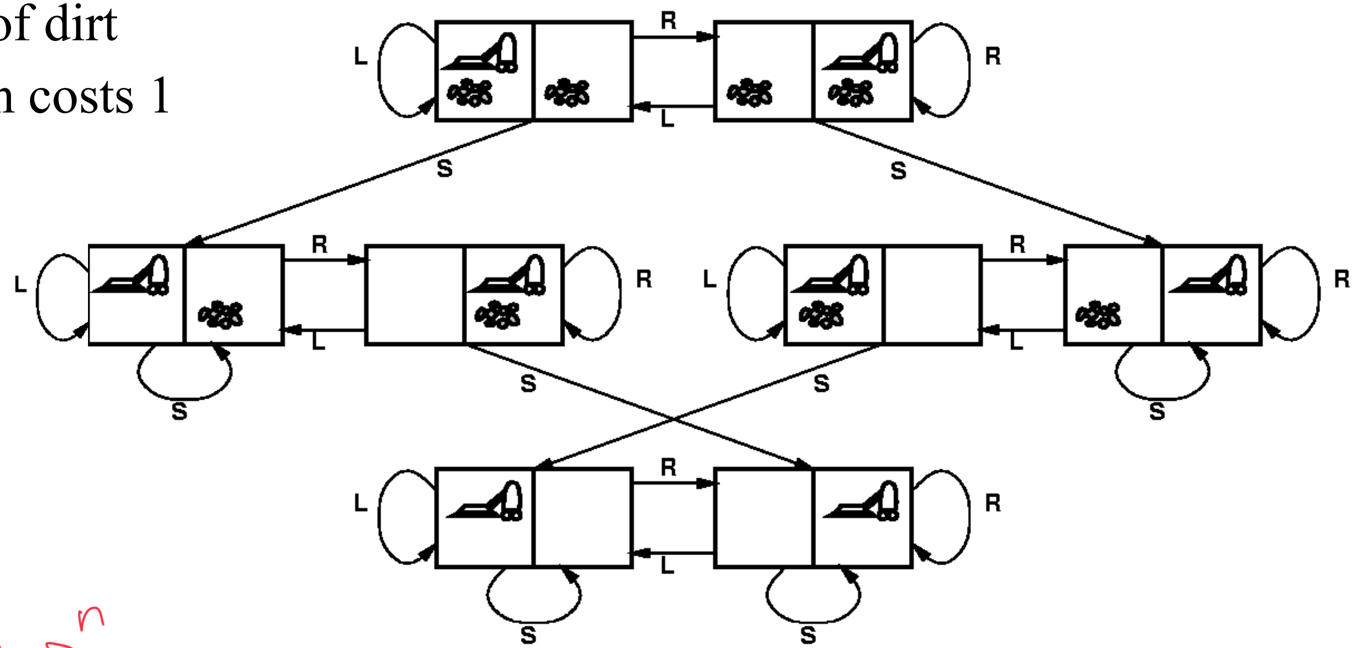
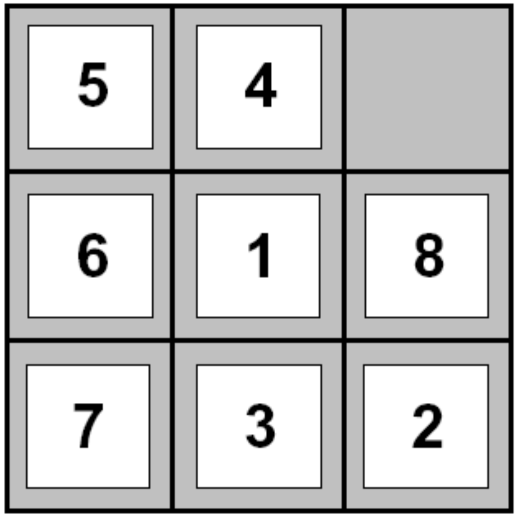


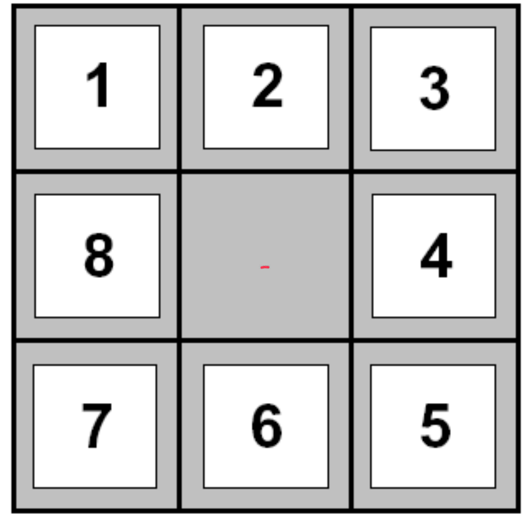
How many states for n grids?

$n \cdot 2^n$

Example Problem: 8-Puzzle



Start State



Goal State

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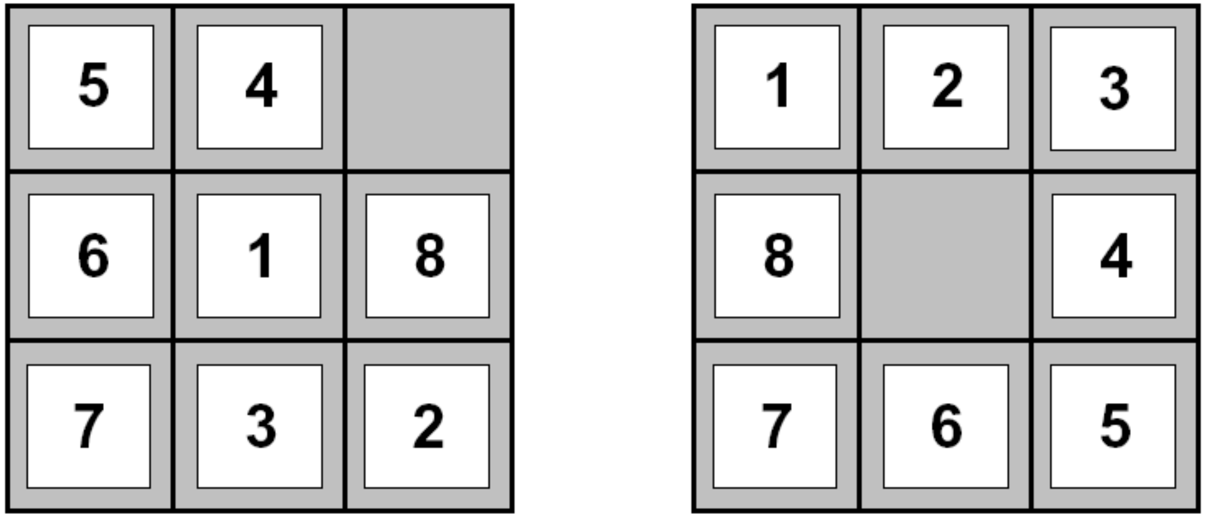


Start State   Goal State

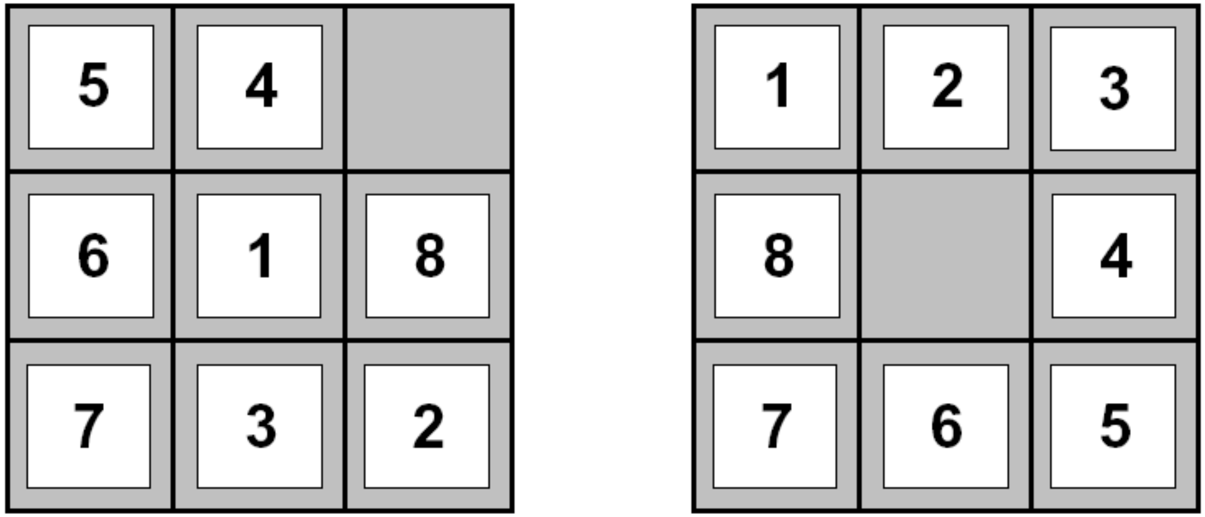
States: Various configurations of the puzzle   9!

Operators: Movements of the blank

Goal test: Goal configuration

Path cost: Each move costs 1

Example Problem: 8-Puzzle



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How many states are there?

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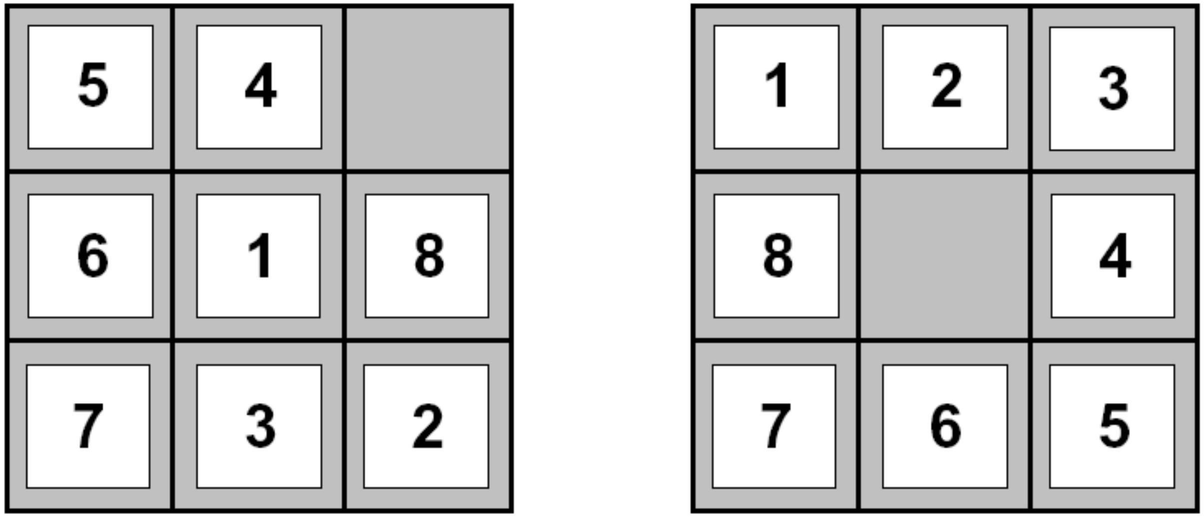

Start State Goal State

States: Various configurations of the puzzle

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Path cost: Each move costs 1

How many states are there?

$9! = 362{,}880$

8-Puzzle is hard (by definition)!

- Optimal solution of the N-puzzle family of problems is NP-complete
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  - Start and end in Bucharest, visit every city at least once
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- Ditto for lots of interesting problems!

Example: Missionaries and Cannibals (3 min discussion)

Problem: Three missionaries and three cannibals are on one side of a river, along with a boat that can hold one or two people. Find a way to get everyone to the other side, without ever leaving a group of missionaries in one place outnumbered by the cannibals in that place

$(3,3,0)$ $S_1 \rightarrow S_2$ $(1,3,1)$

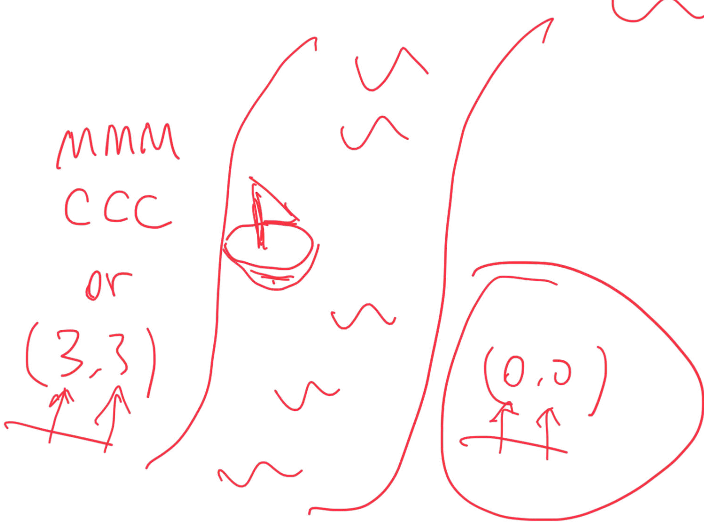
$4 \times 4 \times 2$

(# of M, # of C, binary)

on left

location of the boat

- States, operators, goal test, path cost?



M_1	M_2	M_3
null	null	null

C_1	C_2	C_3

M_1	M_2	M_3	C_1	C_2	C_3
1	0	1	0	0	1

M&C (cont.)

- Initial state
- Goal state

M&C (cont.)

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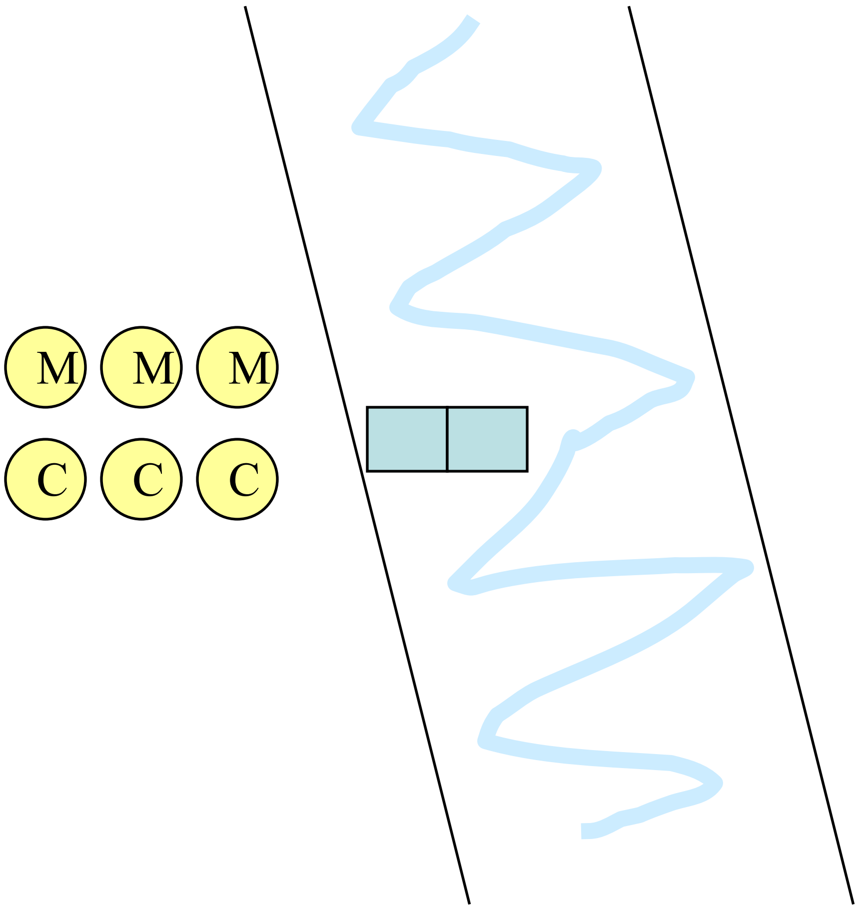
- Goal state



M&C (cont.)

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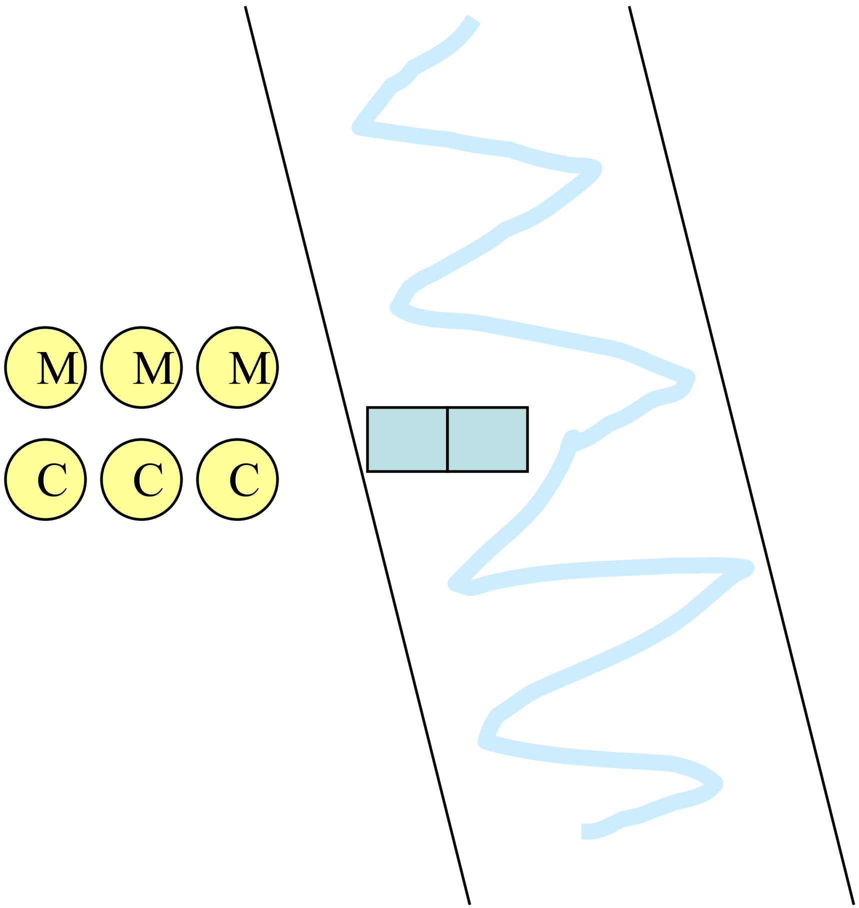
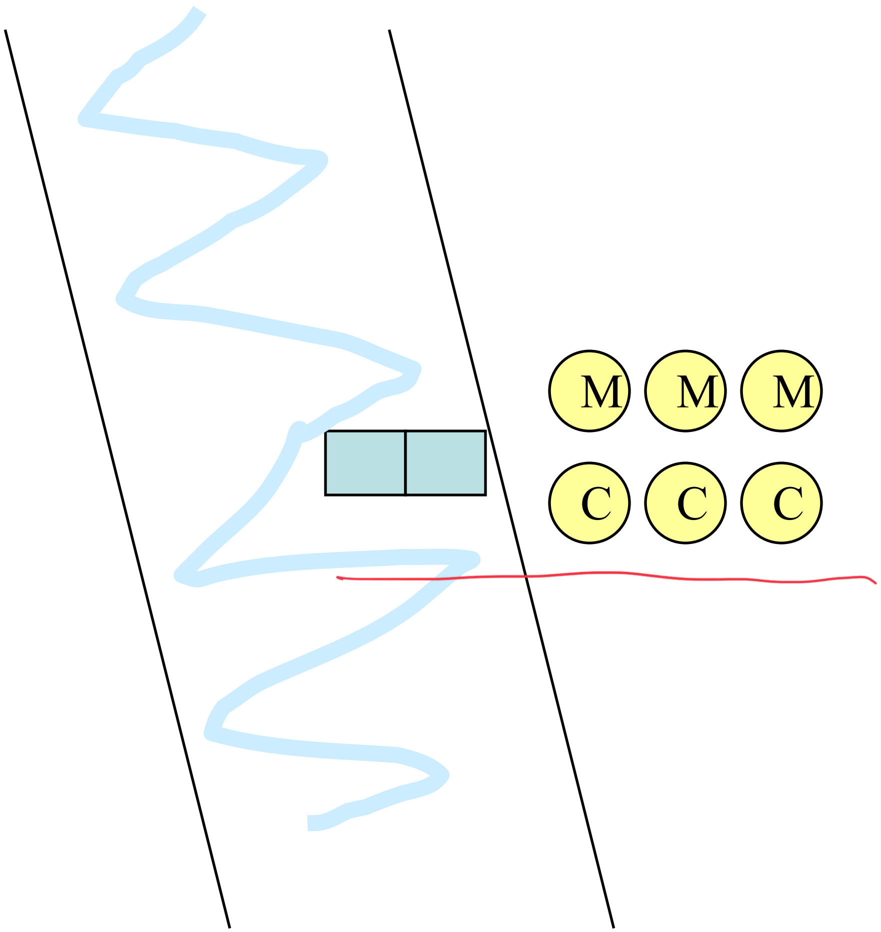


- Goal state



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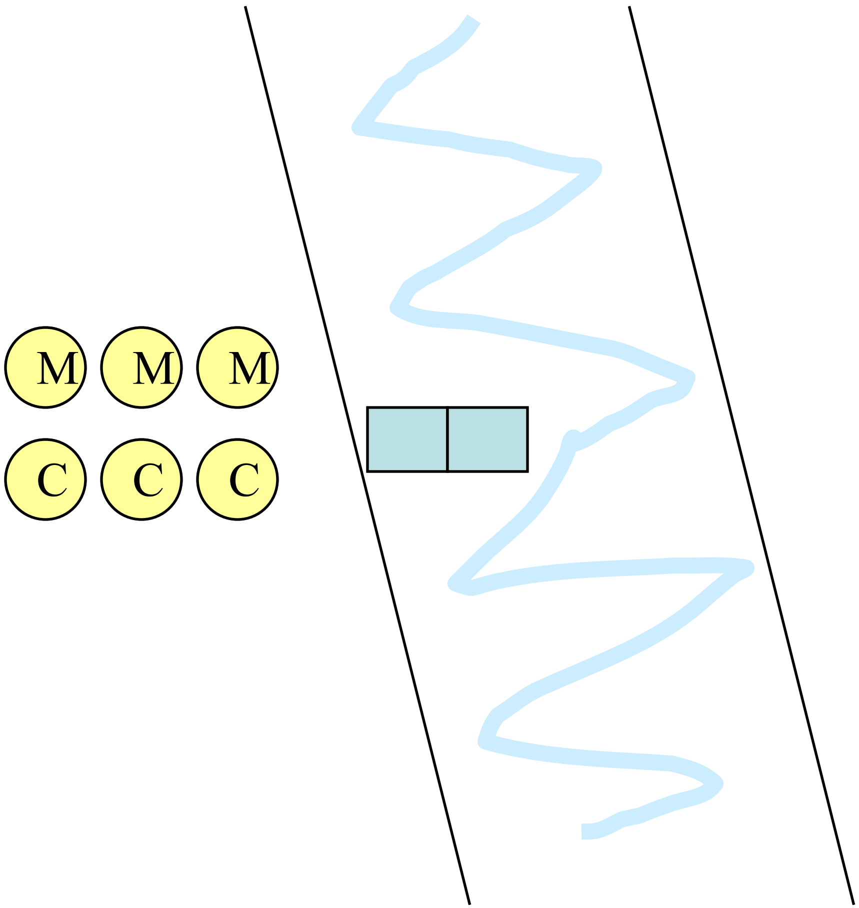

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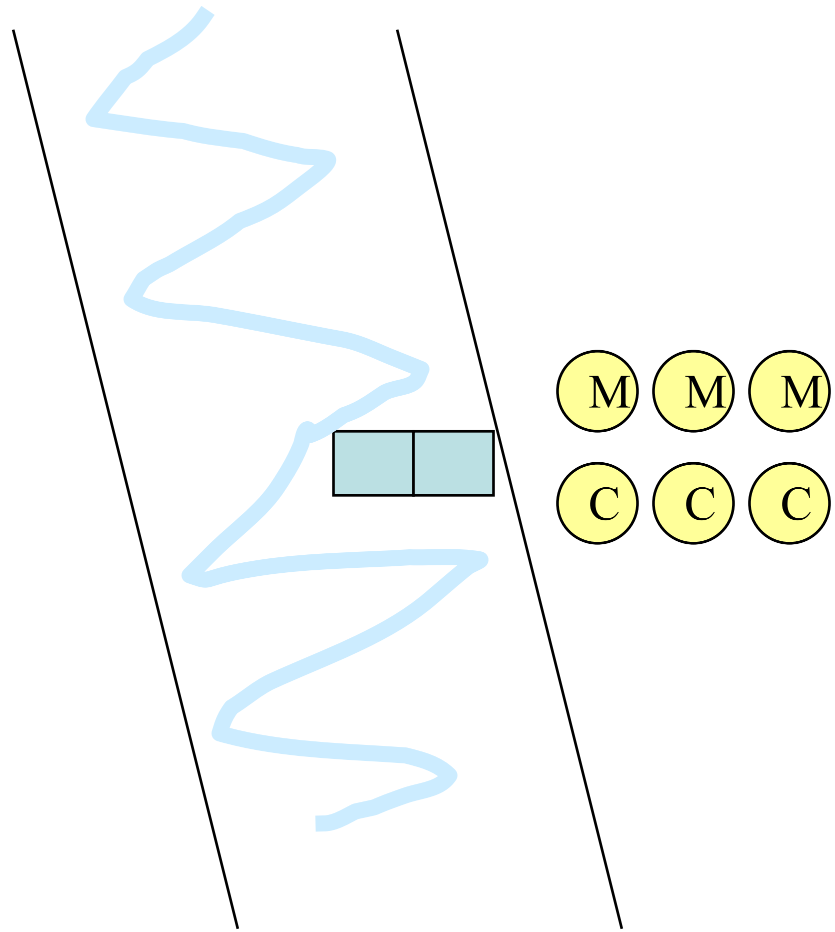
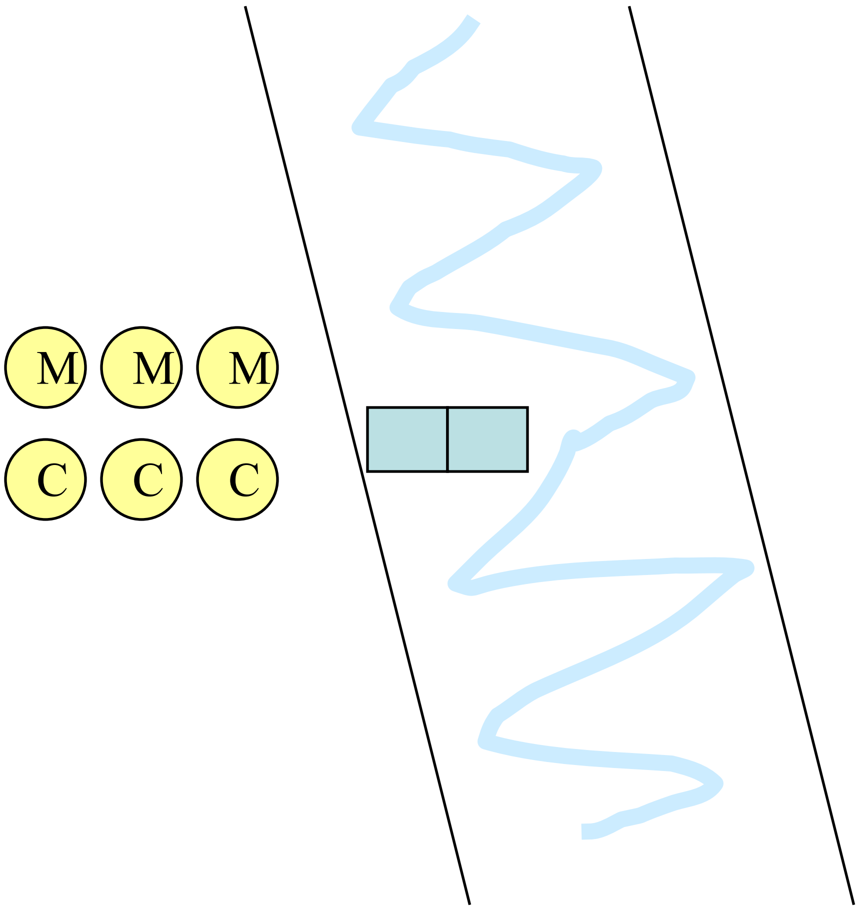


$(M_L \quad C_L \quad B_L)$

M&C (cont.)

- Initial state



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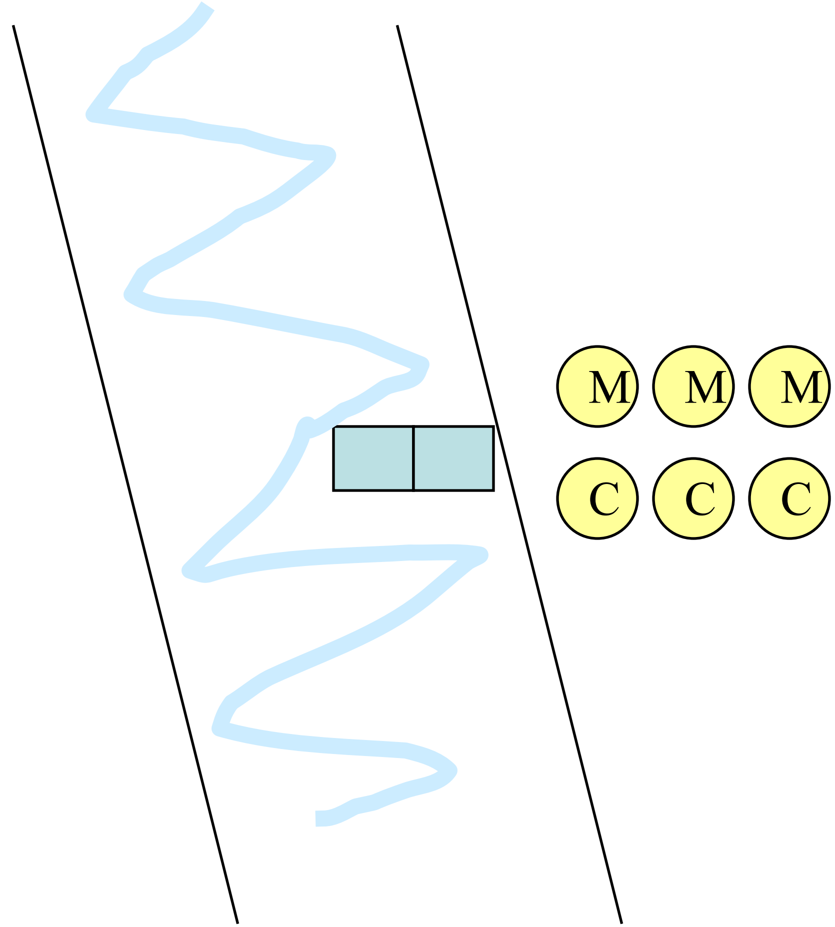

(3 3 1) $(M_L\ C_L\ B_L)$

M&C (cont.)

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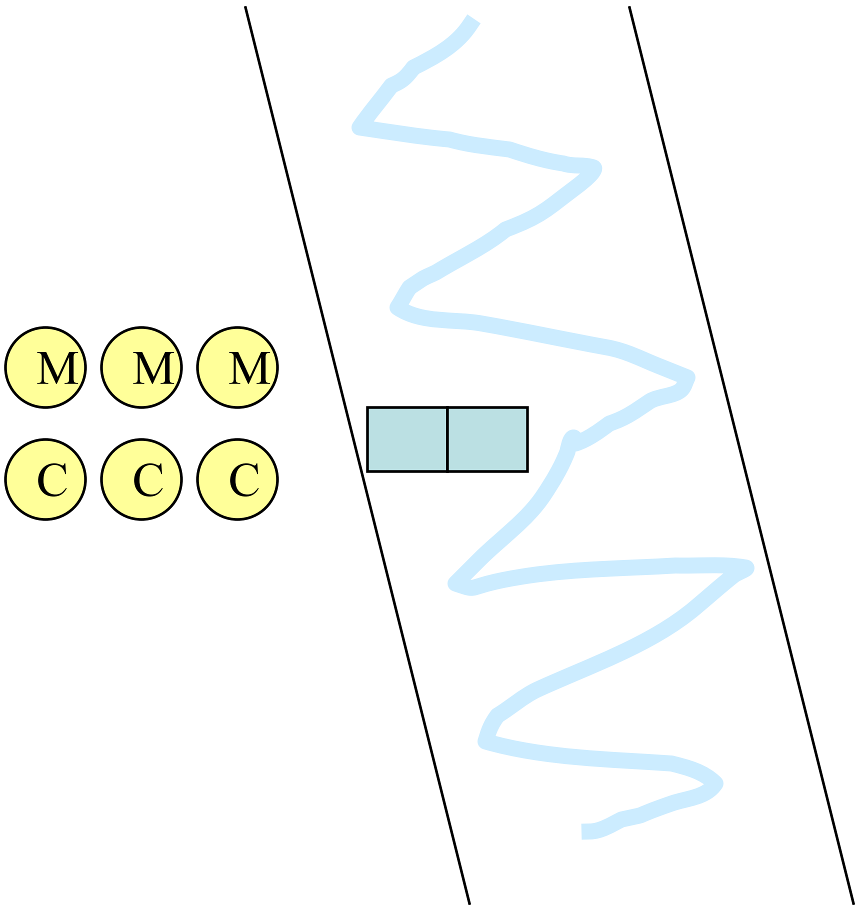

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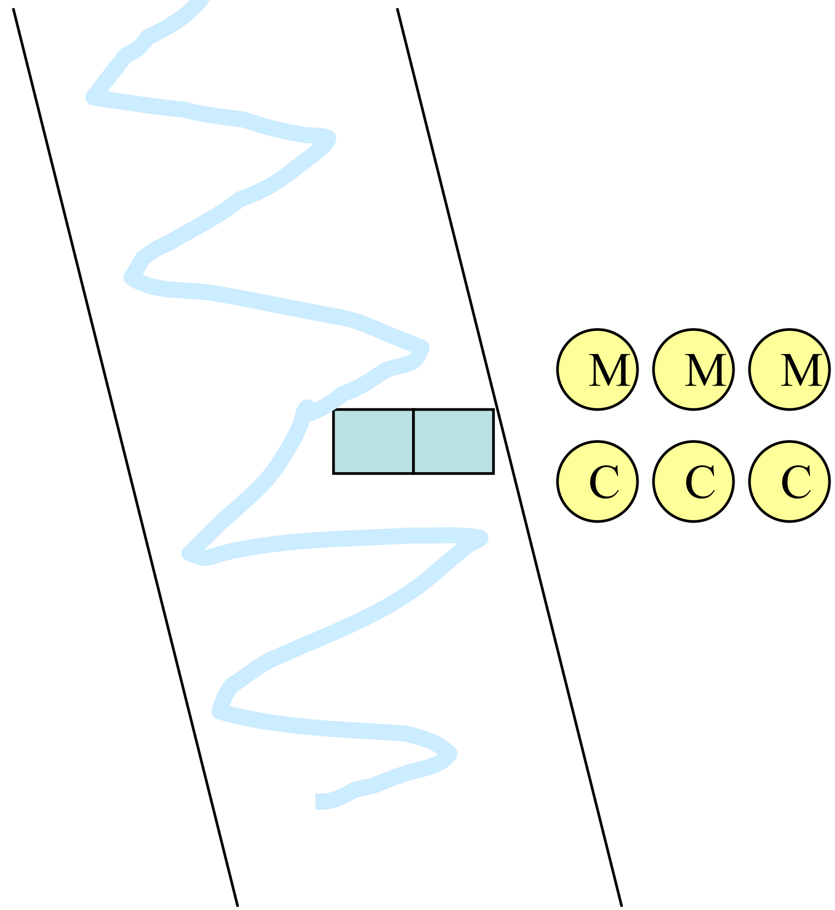
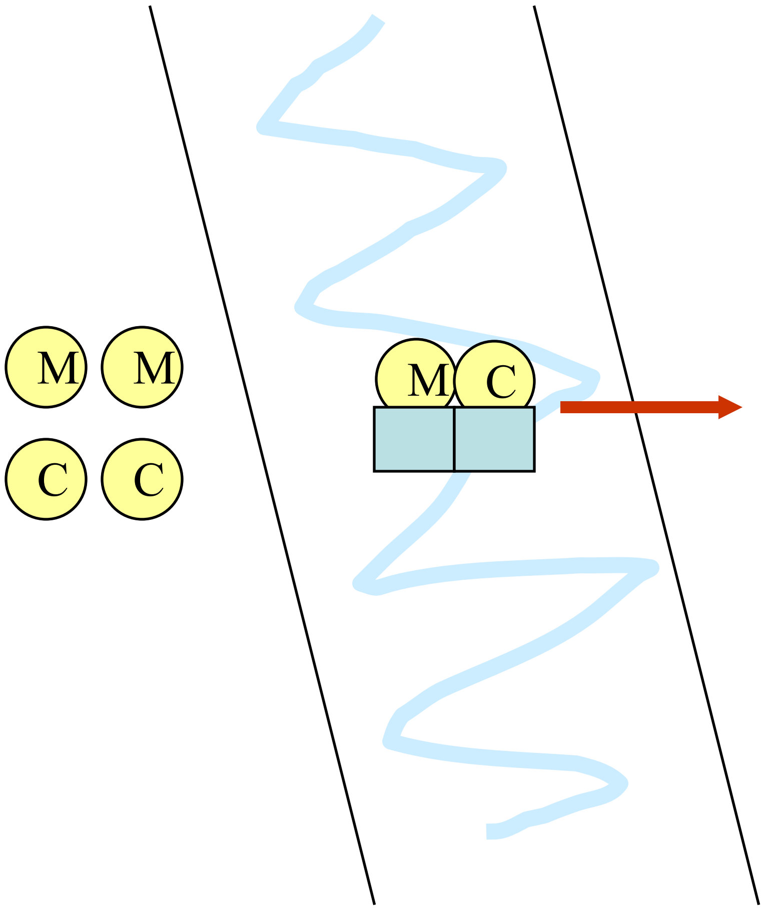


(3 3 1) $(M_L \; C_L \; B_L)$ (0 0 0)

M&C (cont.)



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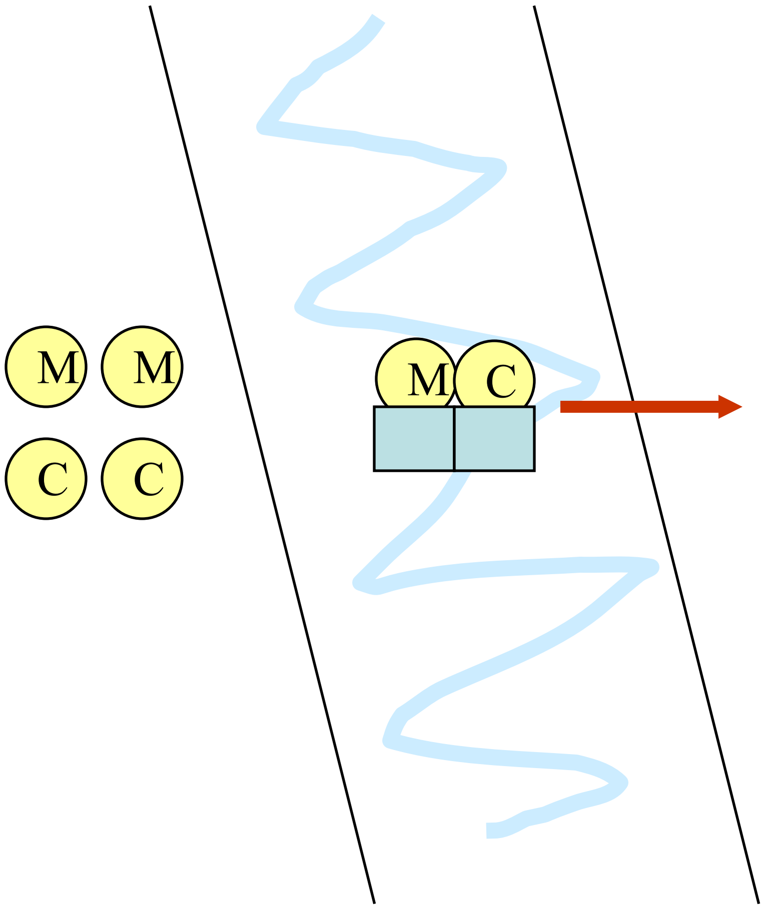

(2 2 0)

M&C (cont.)

- Problem description **<{S}, S_0, {S_{G_j}}, {O_i}, {g_i}>**
- **{S}** : { ({0,1,2,3} {0,1,2,3} {0,1}) }
- **S_0** : (3 3 1)
- **S_G** : (0 0 0)
- **g** = 1
- **{O}** : { (x y b) → (x' y' b') }
- Safe state: (x y b) is safe iff
  - $x > 0$ implies $x \geq y$ and
    $x < 3$ implies $y \geq x$
  - Can be restated as
    $(x = 1 \text{ or } x = 2)$ implies $(x = y)$

Operators:

(x y 1) → (x-2 y 0)
(x y 1) → (x-1 y-1 0)
(x y 1) → (x y-2 0)
(x y 1) → (x-1 y 0)
(x y 1) → (x y-1 0)
(x y 0) → (x+2 y 1)
(x y 0) → (x+1 y+1 1)
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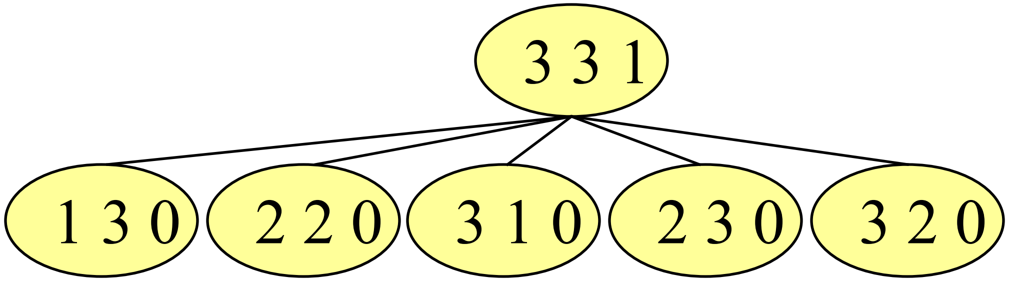
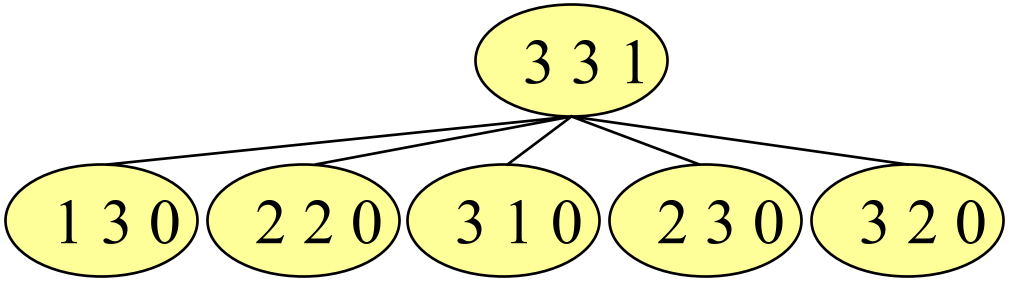
M&C (cont.)



- 11 steps
- 5^{11} = 48 million states to explore

M&C (cont.)



- 11 steps
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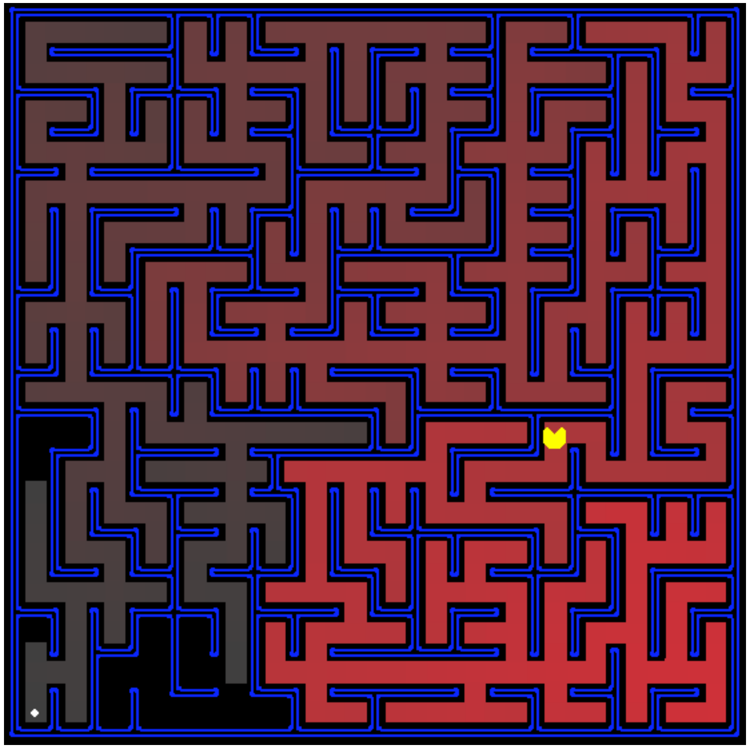
One solution path:

(3 3 1)
(2 2 0)
(3 2 1)
(3 0 0)
(3 1 1)
(1 1 0)
(2 2 1)
(0 2 0)
(0 3 1)
(0 1 0)
(0 2 1)
(0 0 0)

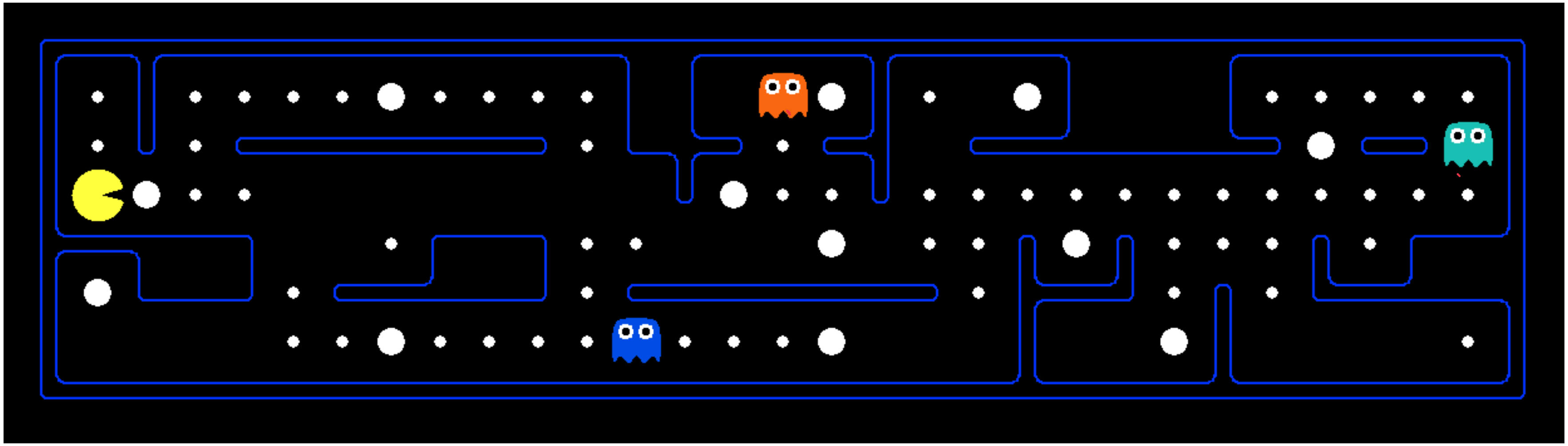
More quizzes: PACMAN

- The goal of a simplified PACMAN is to get to the pellet as quick as possible.
  - For a grid of size 30*30. Everything static.
  - What is a reasonable representation of the State, Operators, Goal test and Path cost?



More quizzes: PACMAN with static ghosts

- The goal is to eat all pellets as quickly as possible while staying alive. Eating the "Power pellet" will allow the pacman to eat the ghost.



- Think about how to formulate this problem. We will revisit it in the next lecture.

Quick summary on problem formulation

- Formulate problems as a search problem
  - Decide your level of abstraction. State, Action, Goal, Cost.
  - Represented by a state-diagram
  - Required solution: A sequence of actions
  - Optimal solution: A sequence of actions with minimum cost.

- Caveats:
  - Might not be a finite graph
  - Might not have a solution
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Let's try solving it anyways!
- Do we need an exact optimal solution?
- Are problems in practice worst case?

Searching for Solutions

- Finding a solution is done by searching through the state space
  - While maintaining a set of partial solution sequences
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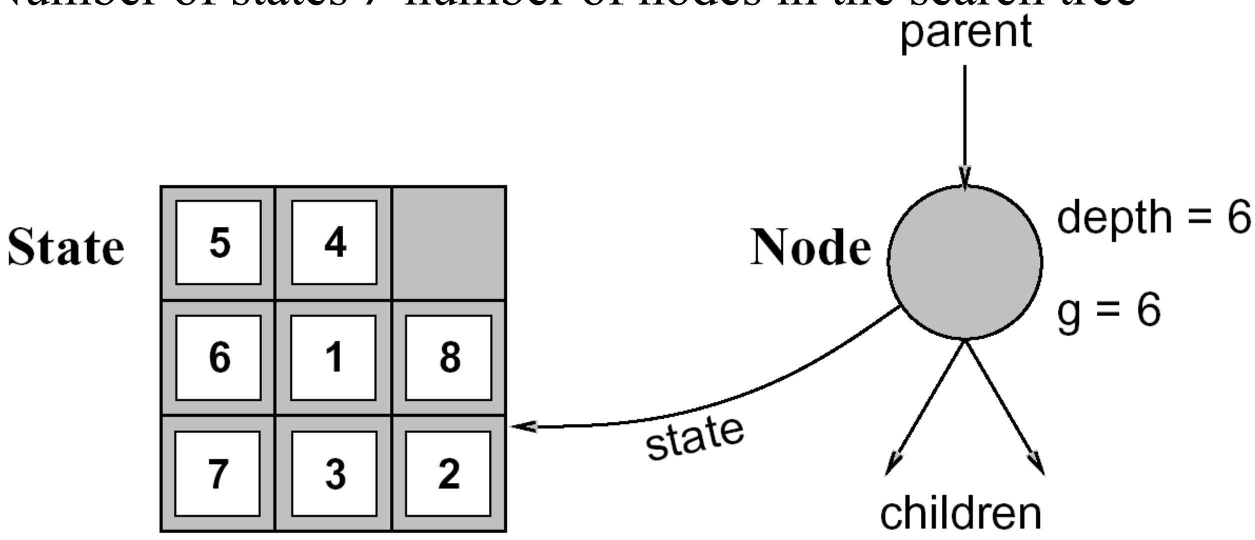
- The *search strategy* determines which states should be expanded first
  - **Expand** a state = Applying the operators to the current state and thereby generating a new set of successor states
- Conceptually, the search process builds up a *search tree* that is superimposed over the state space
  - Root node of the tree ↔ Initial state
  - Leaves of the tree ↔ States to be expanded (or expanded to null)
  - At each step, the search algorithm chooses a leaf to expand

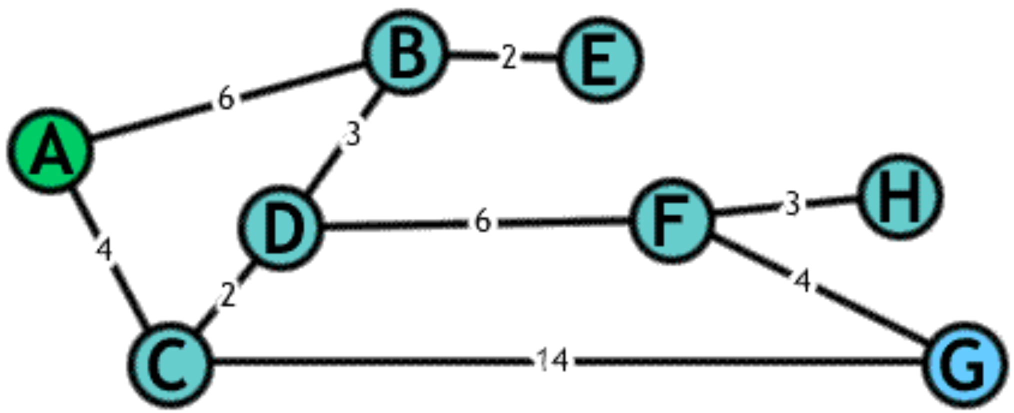
State Space vs. Search Tree

- The **state space** and the **search tree** are not the same thing!
  - A *state* represents a (possibly physical) configuration
  - A *search tree node* is a <u>data structure</u> which includes:
    - { parent, children, depth, path cost }
  - States do not have parents, children, depths, path costs
  - Number of states ≠ number of nodes in the search tree



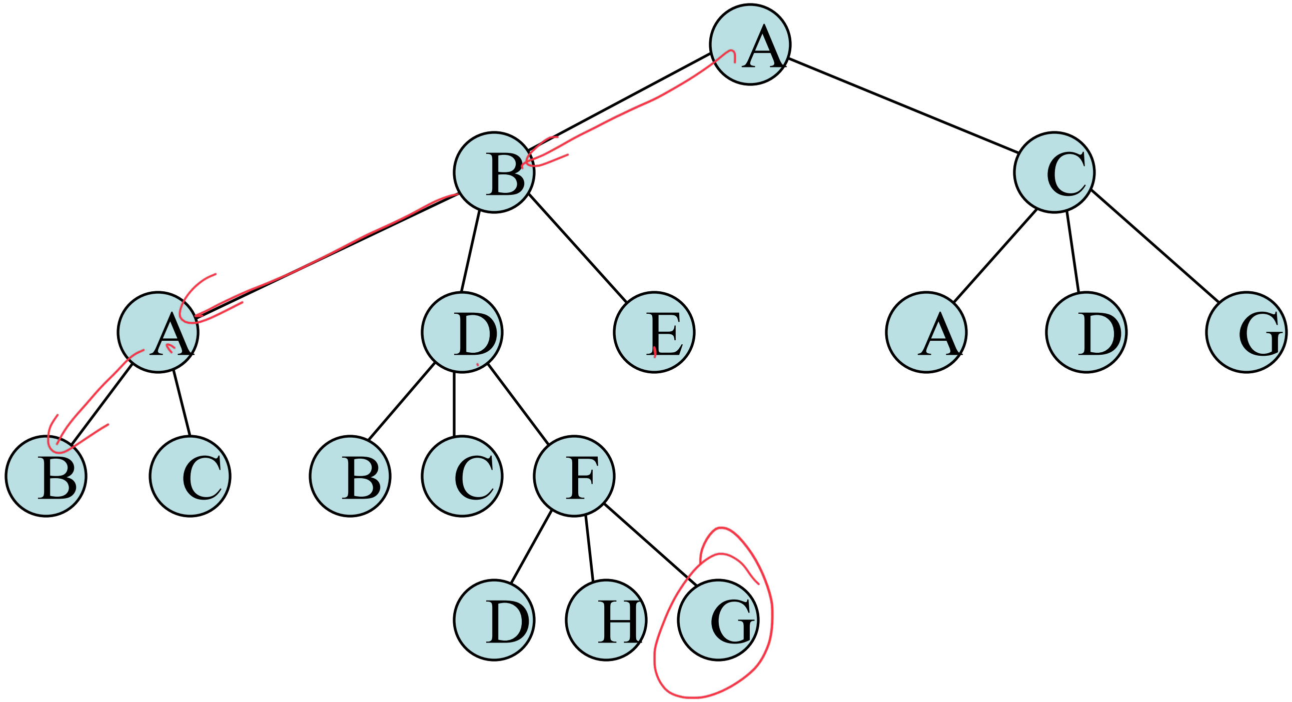
State Space vs. Search Tree (cont.)

State space: 8 states



State Space vs. Search Tree (cont.)

Search tree (partially expanded)



Search Strategies

- Uninformed (blind) search
  - Can only distinguish goal state from non-goal state
- Informed (heuristic) search
  - Can evaluate states

Uninformed (“Blind”) Search Strategies

- No information is available other than
  - The current state
    - Its parent (perhaps complete path from initial state)
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- Blind search strategies
  - Breadth-first search
  - Uniform cost search
  - Depth-first search
  - Depth-limited search
  - Iterative deepening search
  - Bidirectional search

General Search Algorithm (Version 1)

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```
function GENERAL-SEARCH(problem, strategy) returns a solution or failure

 initialize the search tree using the initial state of problem
 loop do
   if there are no candidates for expansion then return failure
   choose a leaf node for expansion according to strategy
   if the node contains a goal state then return the corresponding solution
   else expand the node and add the resulting nodes to the search tree
 end
```

(Called "Tree-Search" in the textbook)

General Search Algorithm (Version 2)

- Uses a queue (a list) and a **queuing function** to implement a *search strategy*
  - **Queuing-Fn(*queue*, *elements*)** inserts a set of elements into the queue and <u>determines the order of node expansion</u>

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function GENERAL-SEARCH(problem, QUEUING-FN) returns a solution or failure

 nodes ← MAKE-QUEUE(MAKE-NODE(INITIAL-STATE[problem]))
 loop do
   if nodes is empty then return failure
   node ← REMOVE-FRONT(nodes)
   if GOAL-TEST[problem] applied to STATE(node) succeeds then return node
   nodes ← QUEUING-FN(nodes, EXPAND(node, OPERATORS[problem]))
 end
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  end
```

"Nodes" is also known as a **"frontier"** --- the set of states Nodes we haven't yet explored/expanded.
"EXPAND" is known as the "**successor function**" --- the set of all states that you could expand on.

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- Primary criteria to evaluate search strategies
  - **Completeness**
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  - b = Maximum branching factor of the search tree
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  - O(b^3d^2) – polynomial time
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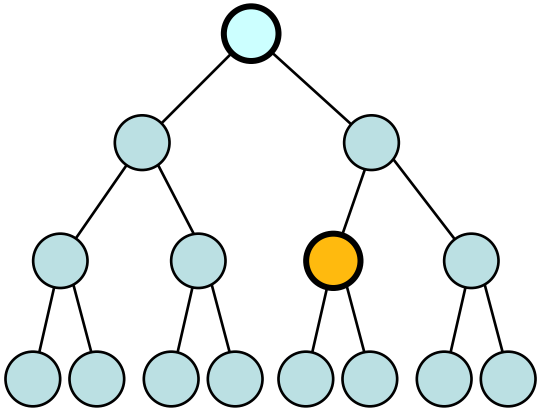
Search tree



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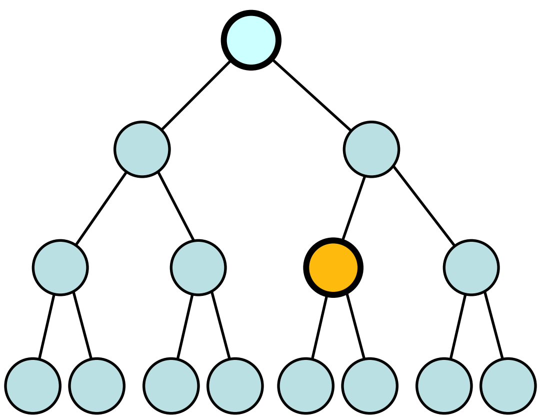

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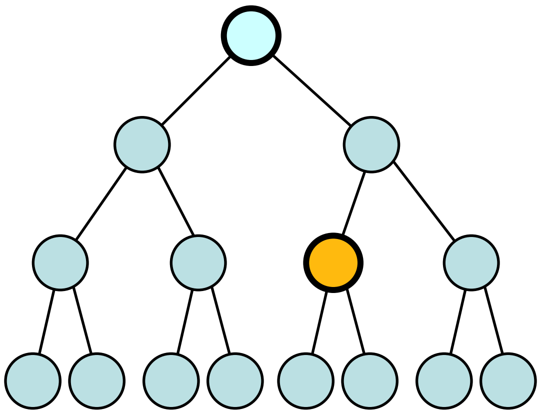

For chess, b_{ave}= 35

$b = 2, \quad d = 2, \quad m = 3$

Breadth-First Search

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- All nodes at depth *d* in the search tree are expanded before any nodes at depth *d+1*
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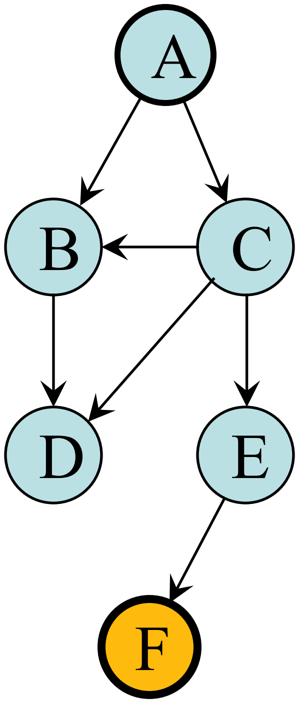
function BREADTH-FIRST-SEARCH(*problem*) **returns** a solution or failure
 return GENERAL-SEARCH(*problem,* ENQUEUE-AT-END)

Example

State space graph



Search tree

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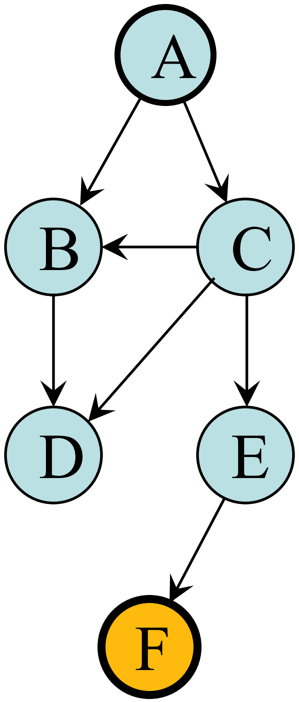

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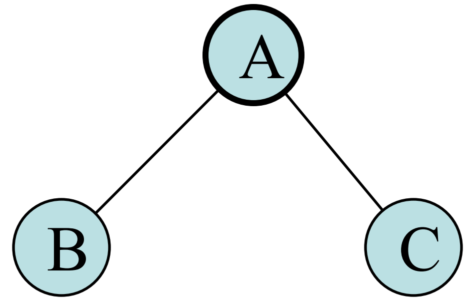

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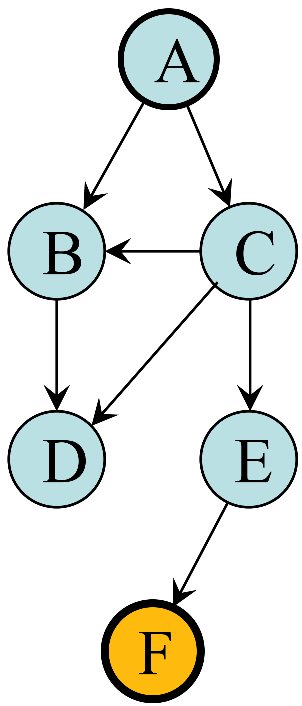

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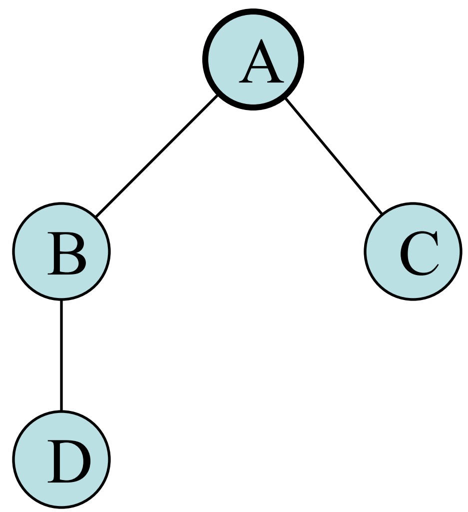

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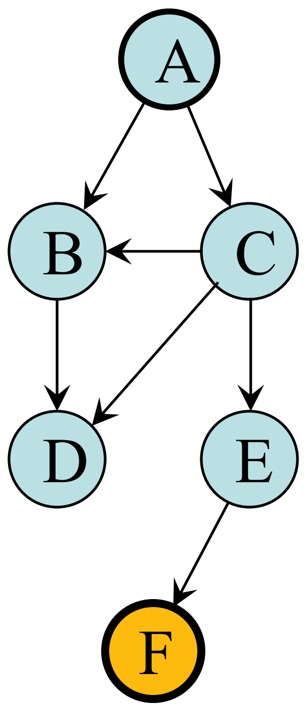

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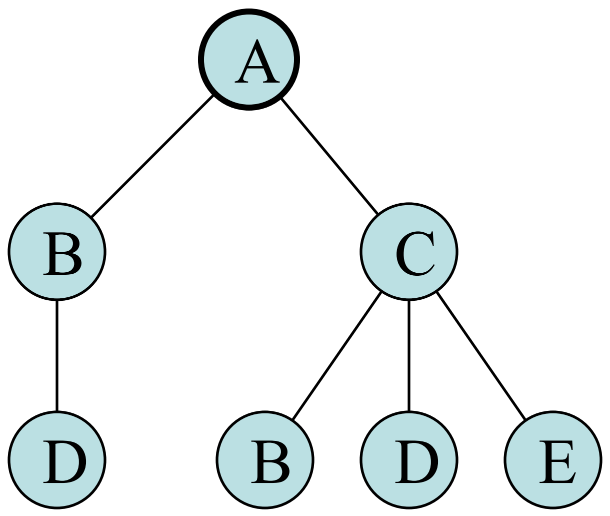

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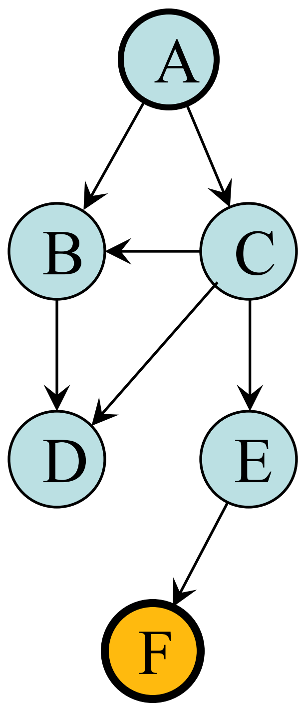

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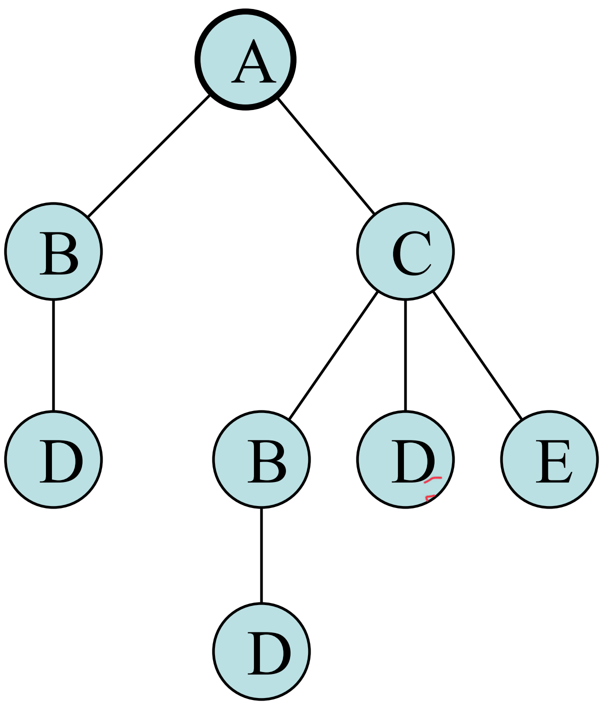

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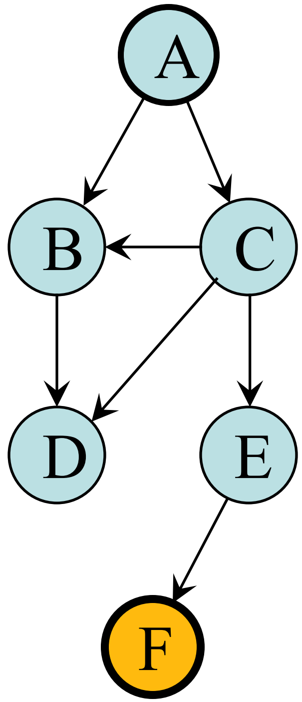

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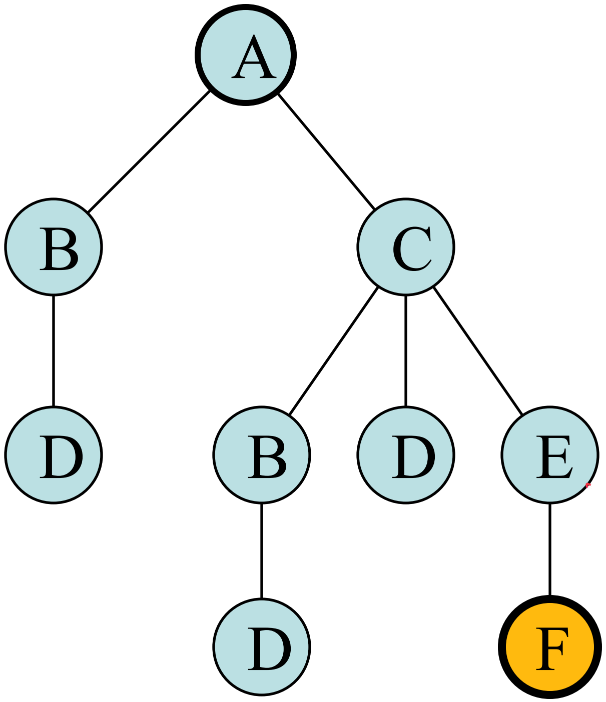

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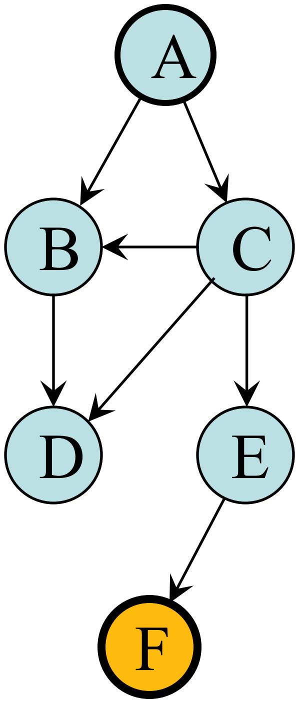

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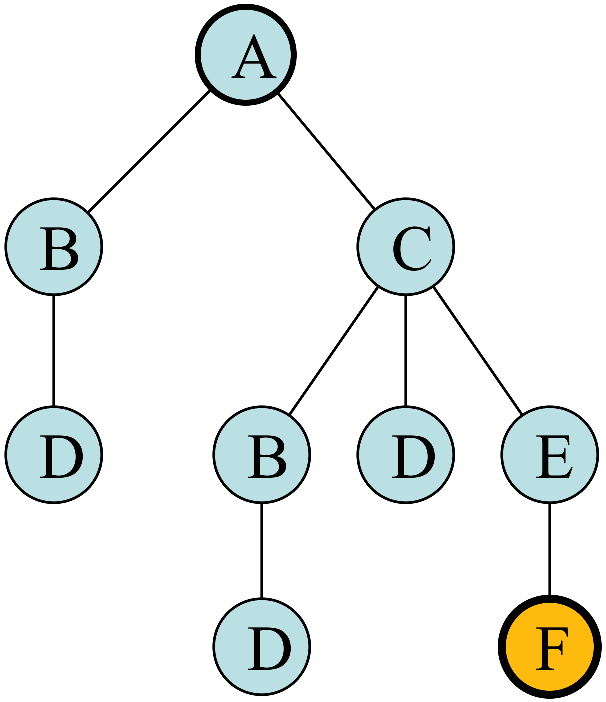
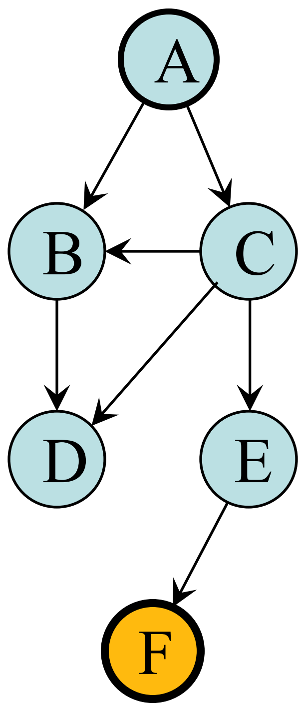


Queue

Example

State space graph



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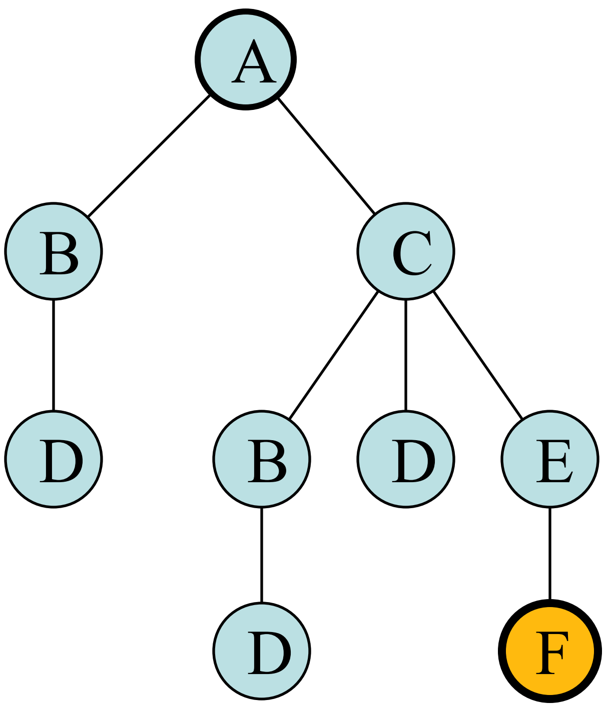
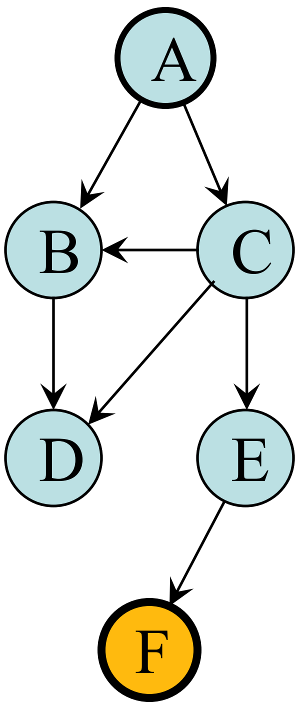


Queue

(A)

Example

State space graph



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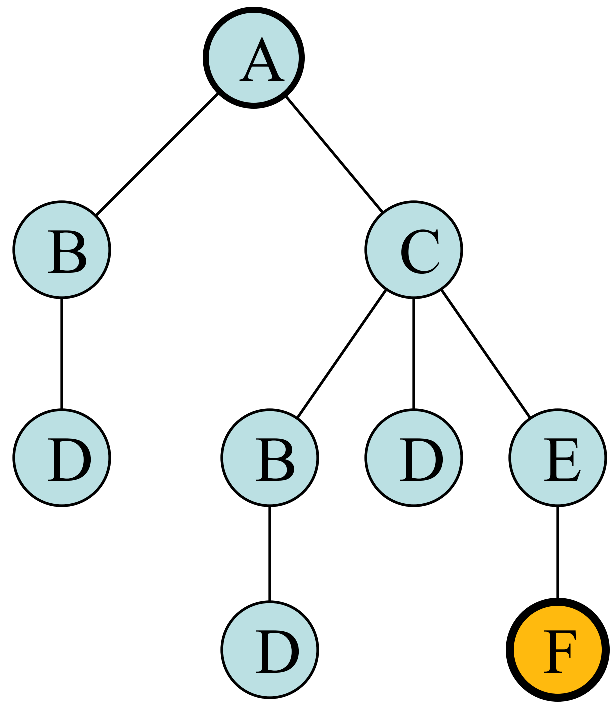


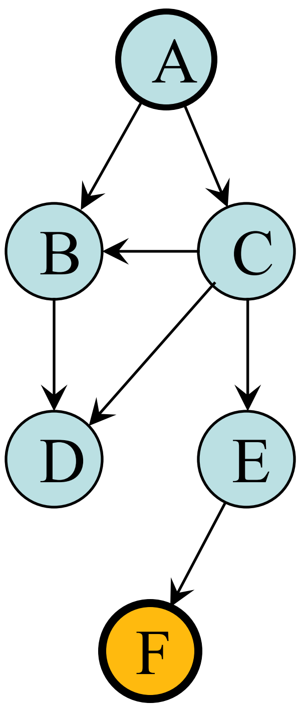
Queue

(A)

(B C)

Example

State space graph



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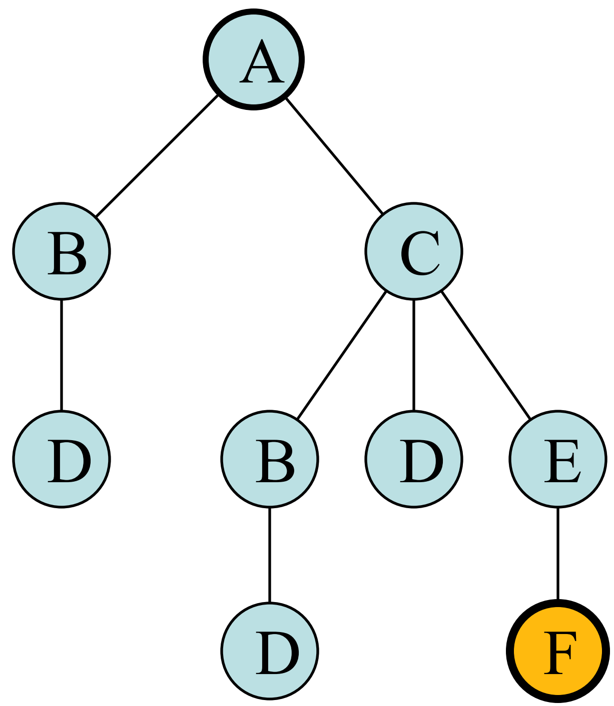

Queue

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(B C)

(C D)

Example

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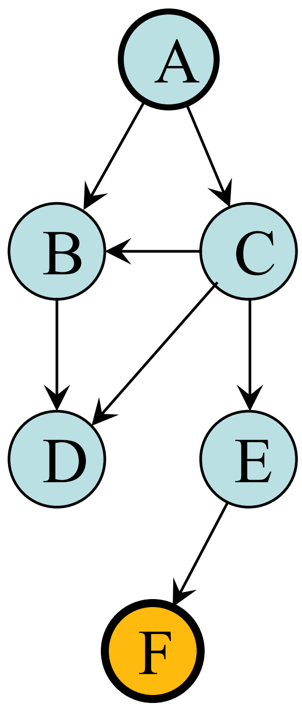
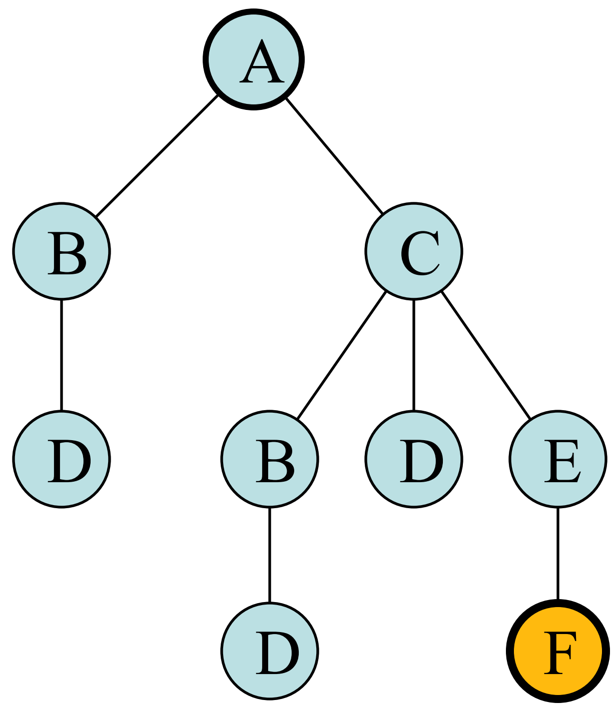


Search tree



Queue

(A)

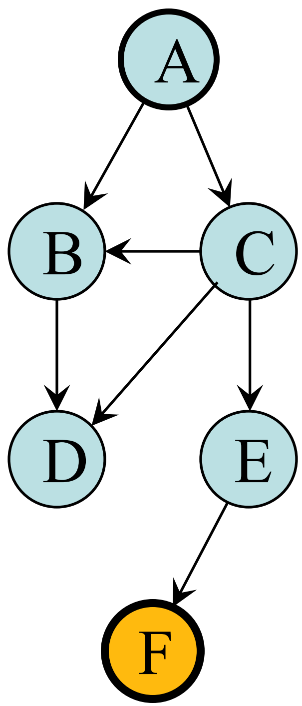
(B C)

(C D)

(D B D E)

Example

State space graph



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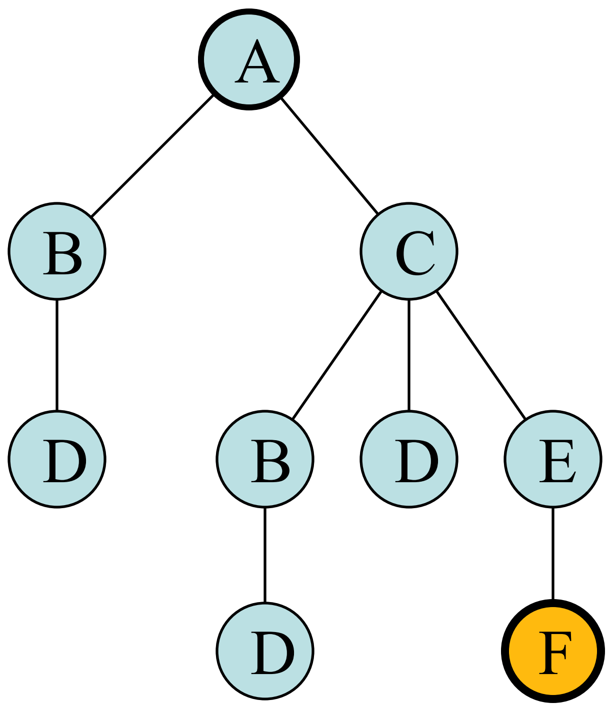

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Example

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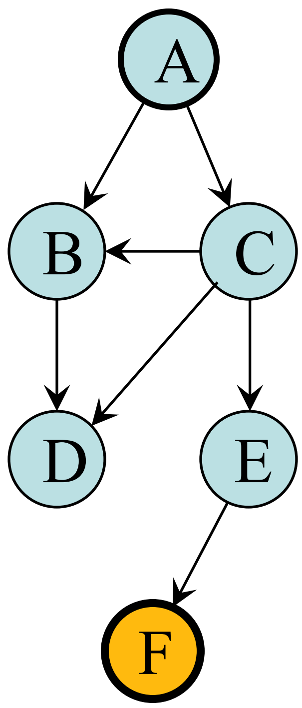

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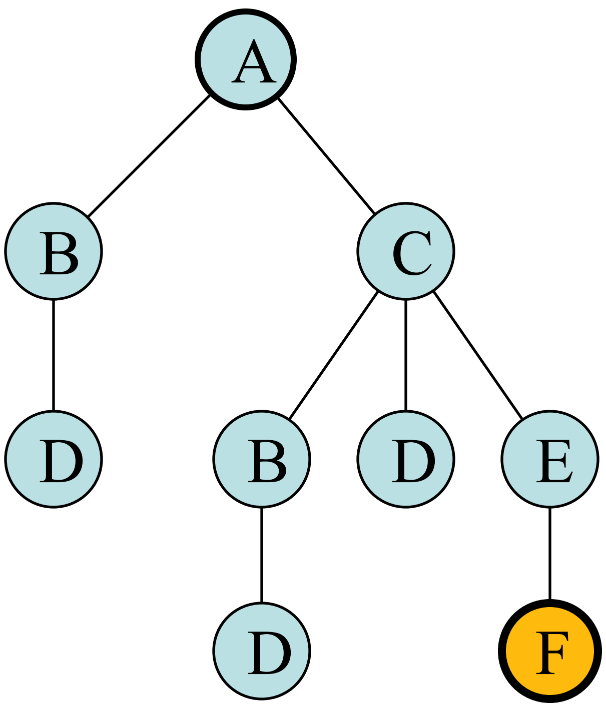

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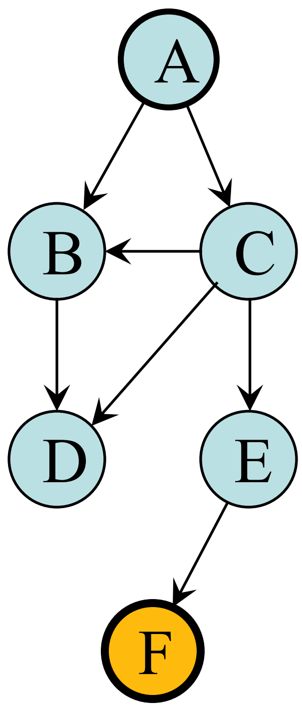
Example

State space graph



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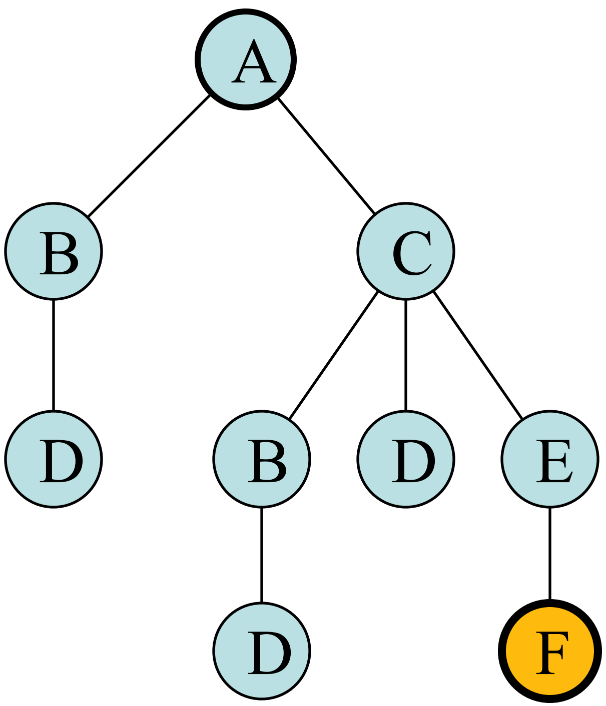

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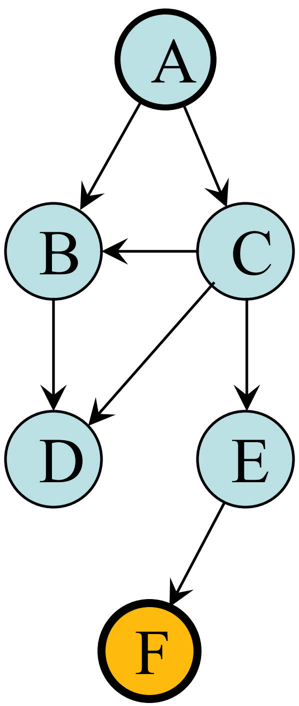
State space graph



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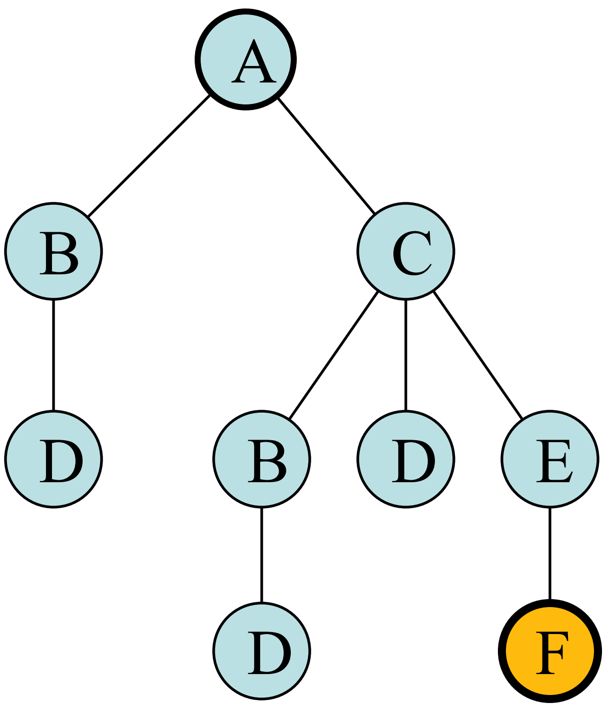

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Example

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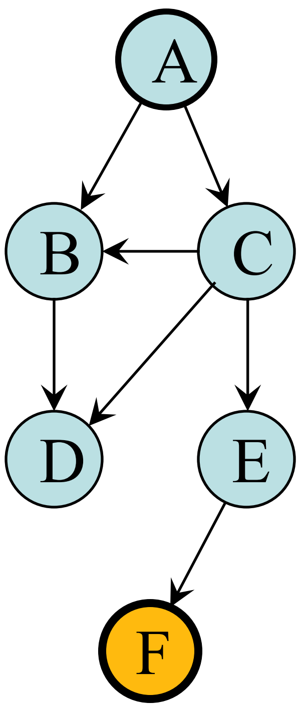

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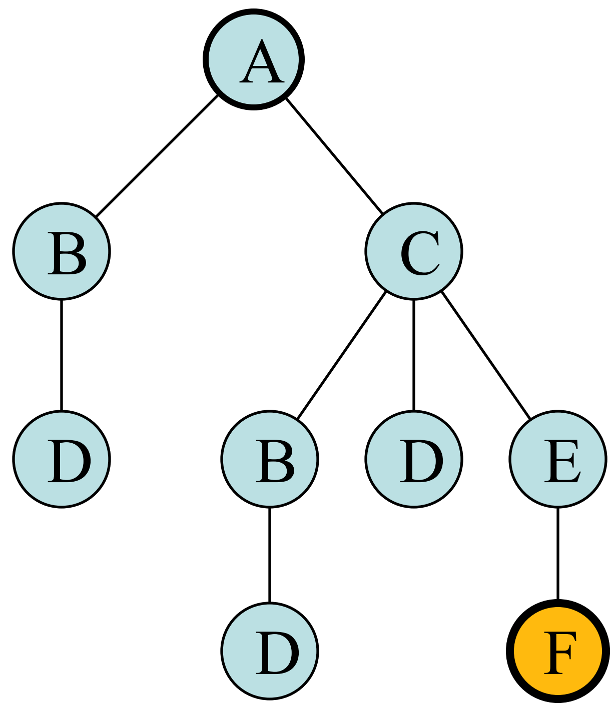

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Example

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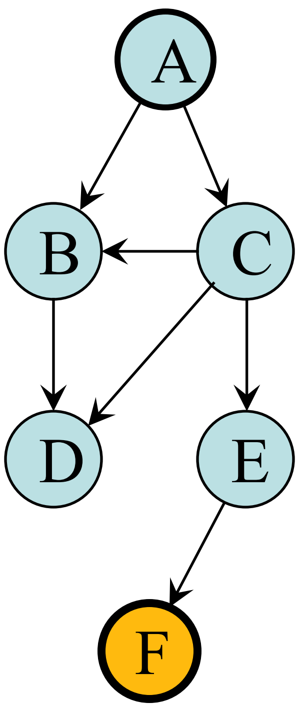

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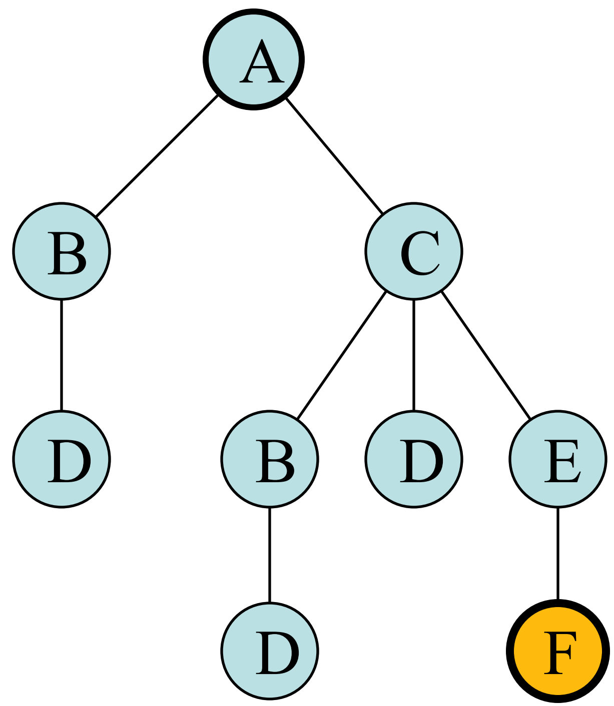

Queue

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Breadth-First Search

- Complete?
- Optimal?
- Time complexity?
- Space complexity?

b = branching factor (require finite b)
d = depth of shallowest solution

Breadth-First Search

- Complete? Yes
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Breadth-First Search

- Complete? Yes
- Optimal? If shallowest goal is optimal
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Breadth-First Search

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- Time complexity? Exponential: **O(** $\boldsymbol{b^{d+1}}$ **)**
- Space complexity?

b = branching factor (require finite b)
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Breadth-First Search

- Complete? Yes
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Breadth-First Search

- Complete? Yes
- Optimal? If shallowest goal is optimal
- Time complexity? Exponential: **O(** b^{d+1} **)**
- Space complexity? Exponential: **O(** b^{d+1} **)**

In practice, the memory requirements are typically worse than the time requirements

b = branching factor (require finite b)
d = depth of shallowest solution

Depth-First Search

Depth-First Search

- Always expands one of the nodes at the deepest level of the tree
  - Low memory requirements
  - Problem: depth could be infinite
- Uses a stack (LIFO)

Depth-First Search

- Always expands one of the nodes at the deepest level of the tree
  - Low memory requirements
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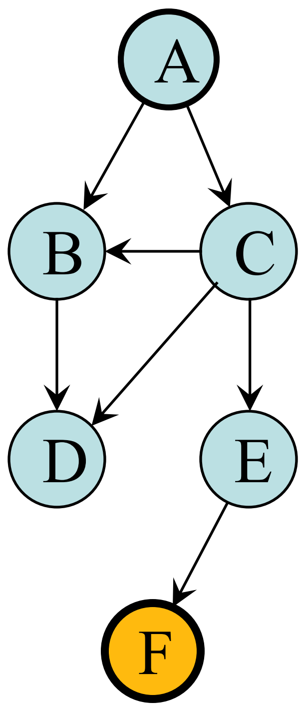
- Uses a stack (LIFO)

function DEPTH-FIRST-SEARCH(*problem*) **returns** a solution or failure
return **GENERAL-SEARCH**(*problem*, ENQUEUE-AT-FRONT)

Example

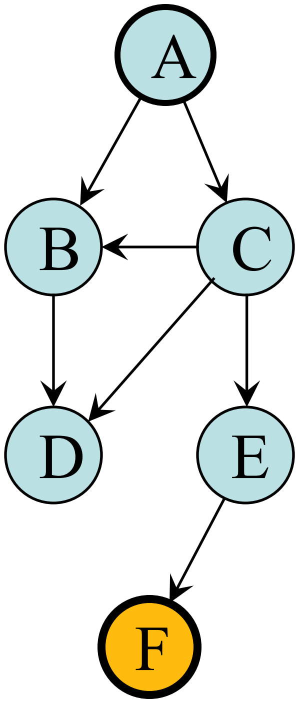
State space graph



Search tree

Example

State space graph



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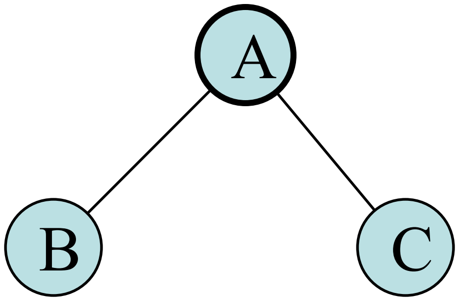

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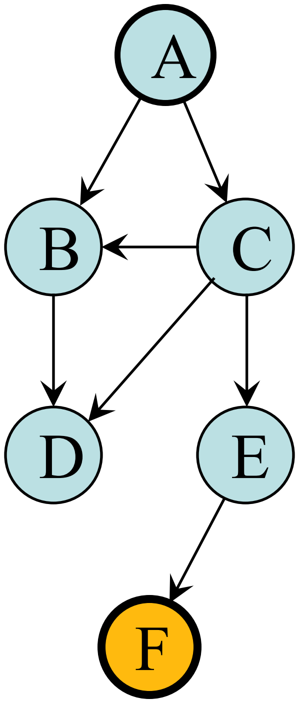

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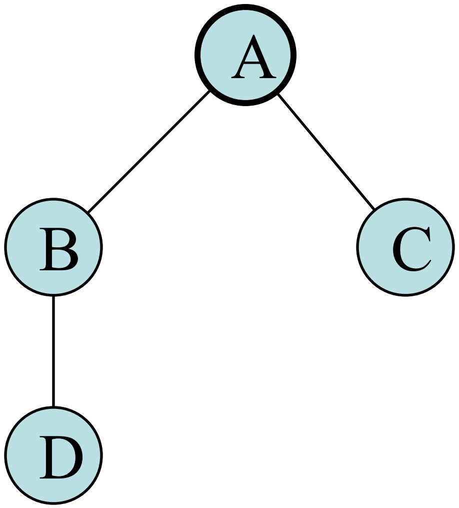

Example

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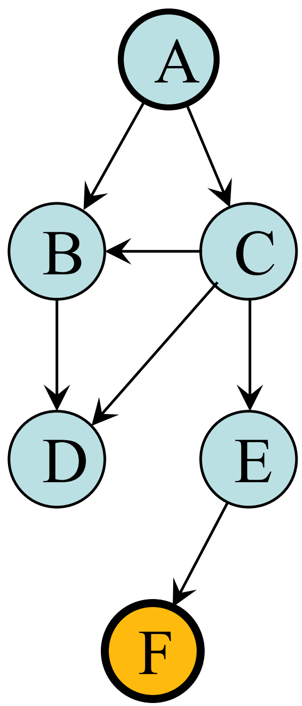

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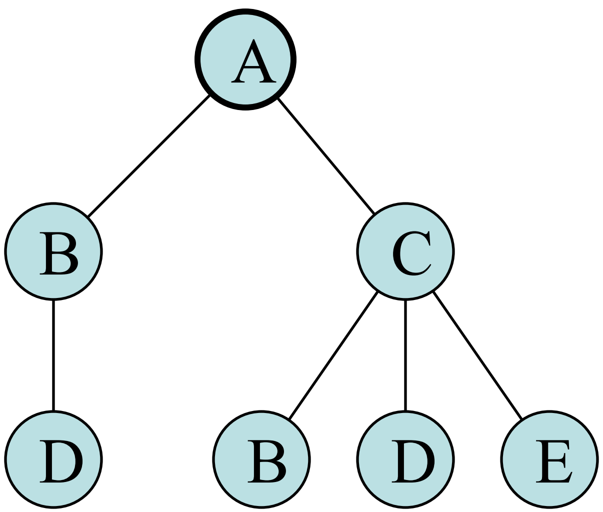

Example

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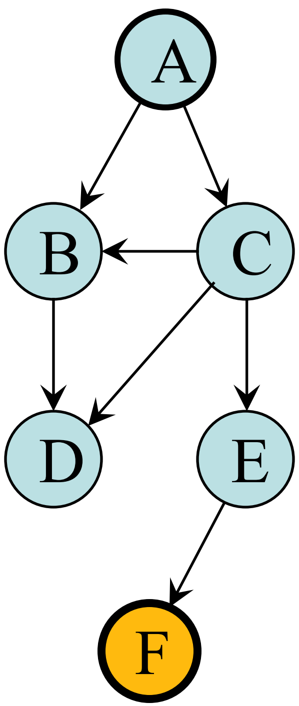

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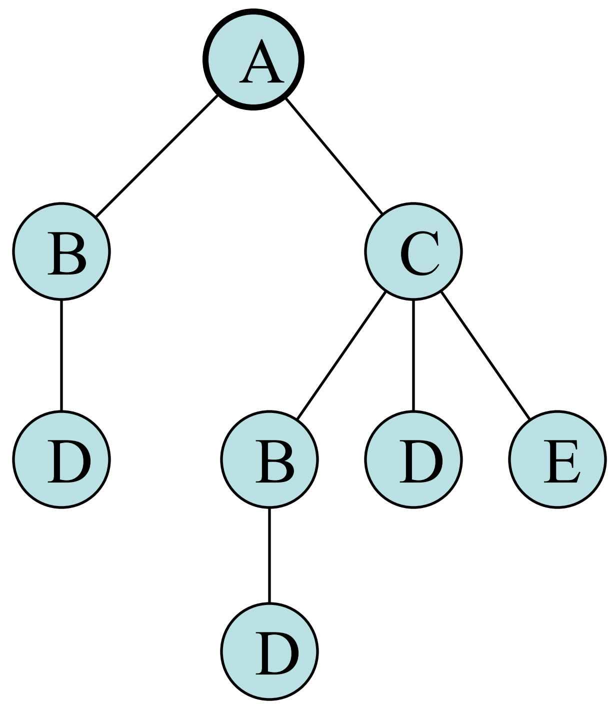

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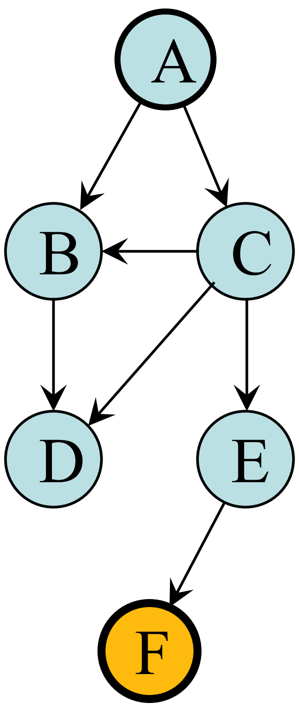

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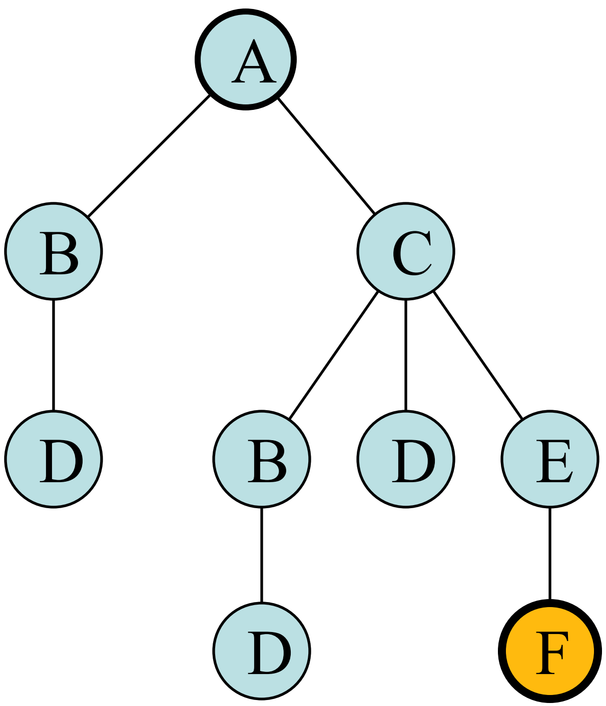

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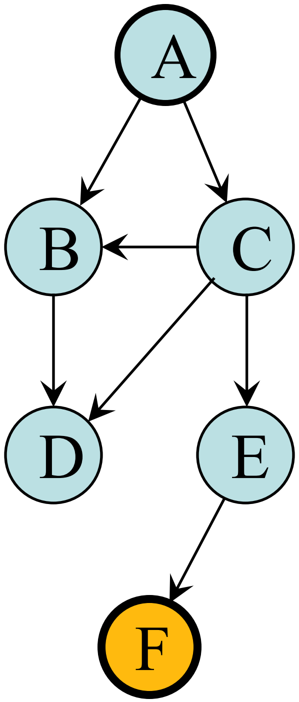

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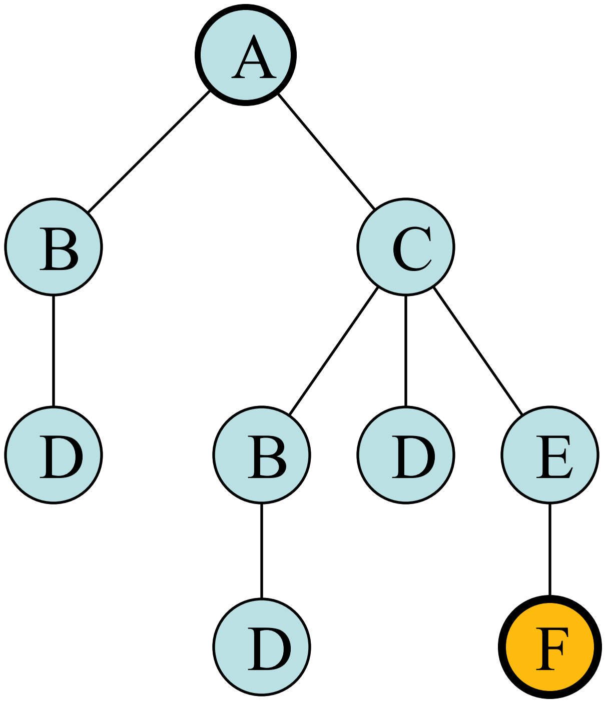
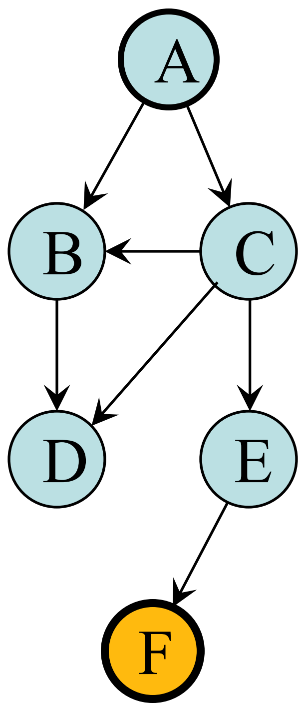


Queue

Example

State space graph



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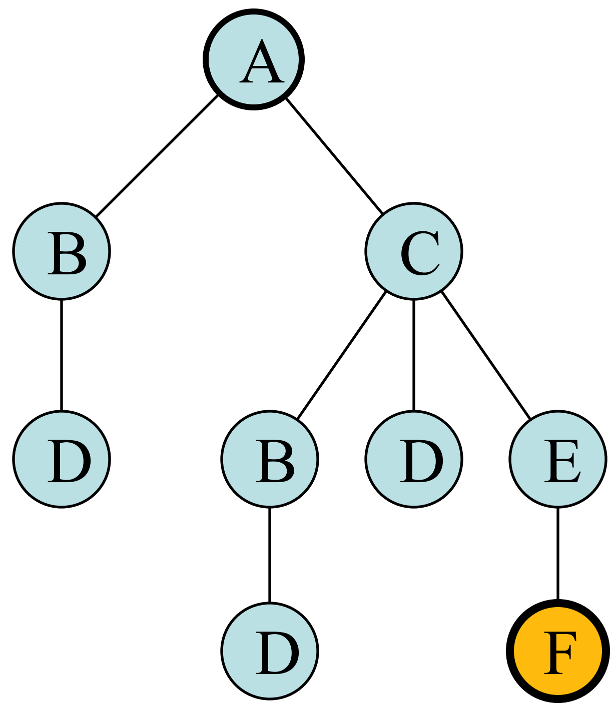
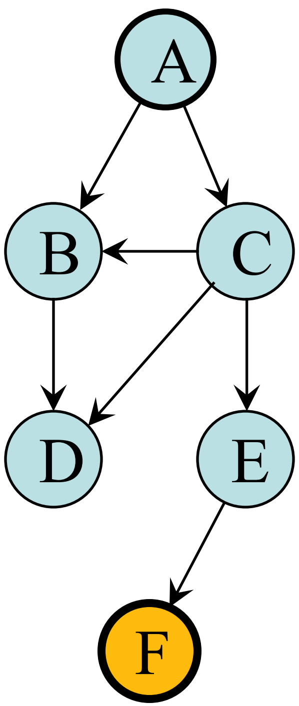


Queue

(A)

Example

State space graph



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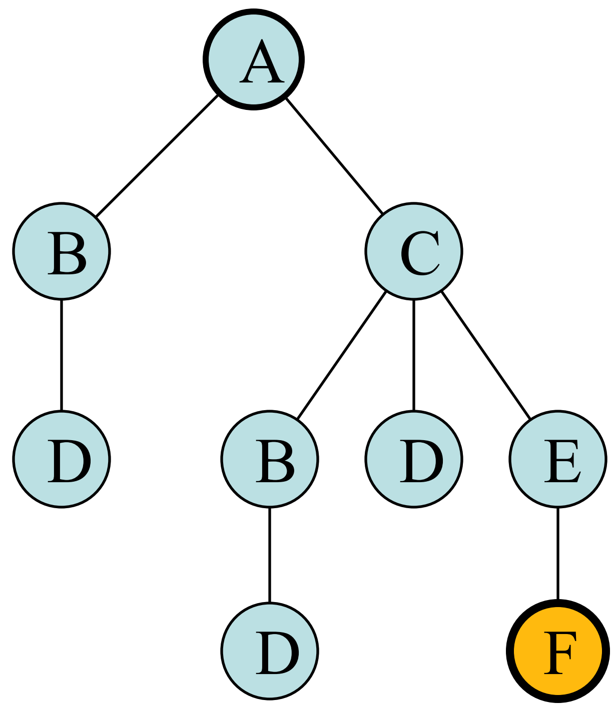


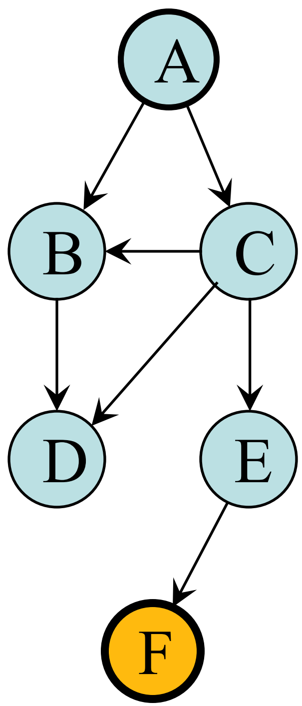
Queue

(A)

(B C)

Example

State space graph



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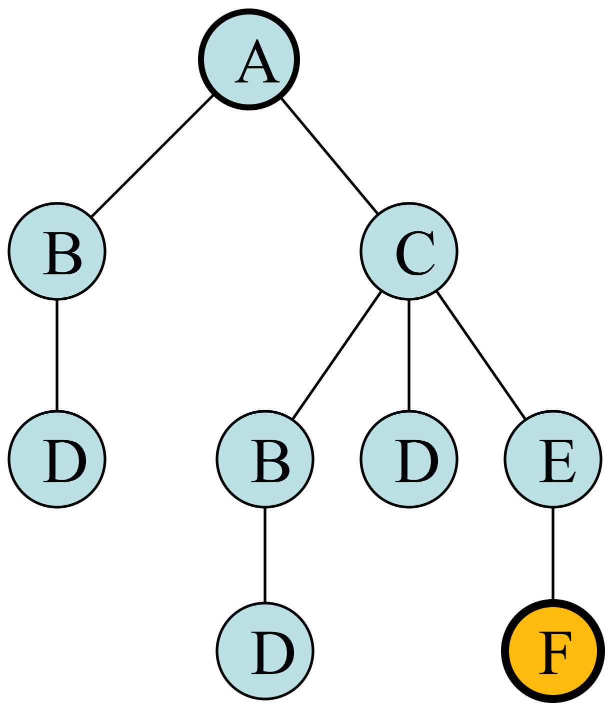

Queue

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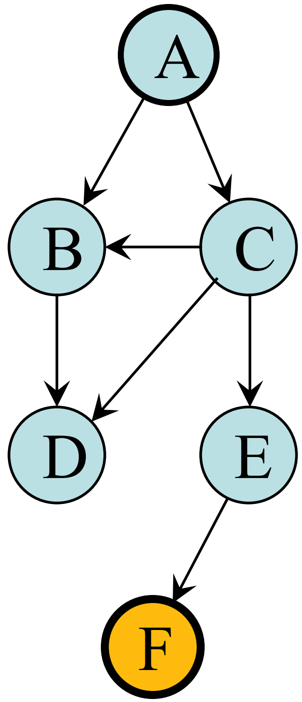
Example

State space graph



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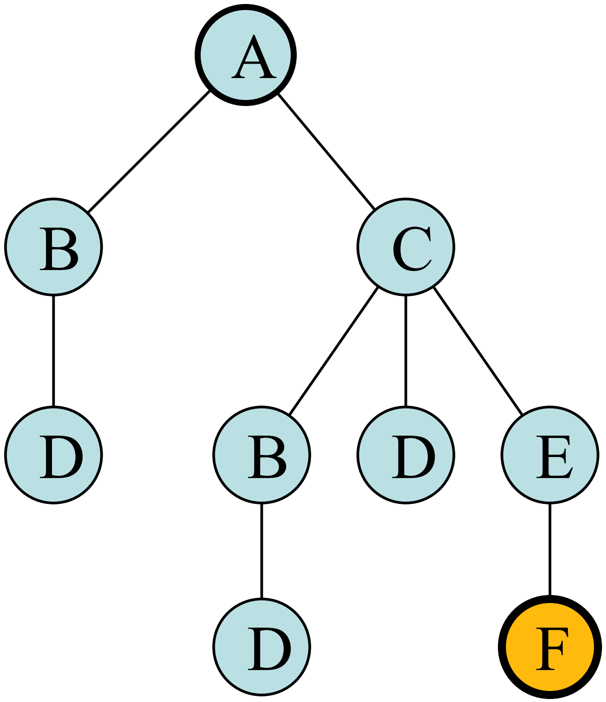

Queue

(A)

(B C)

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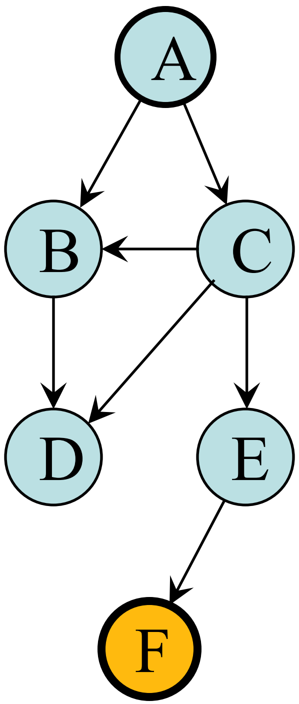
(C)

Example

State space graph



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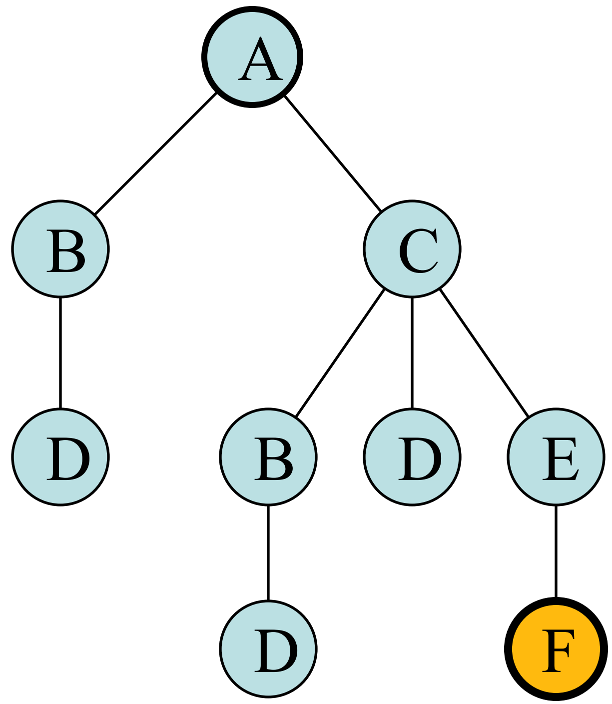

Queue

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Example

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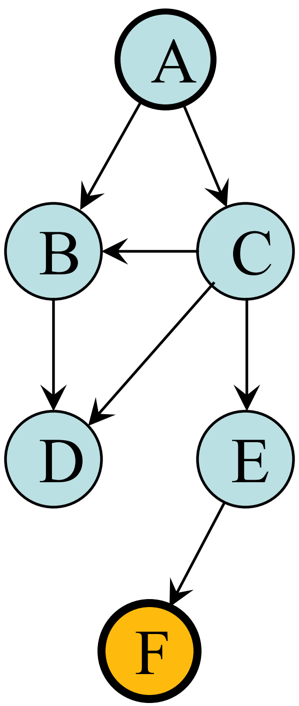

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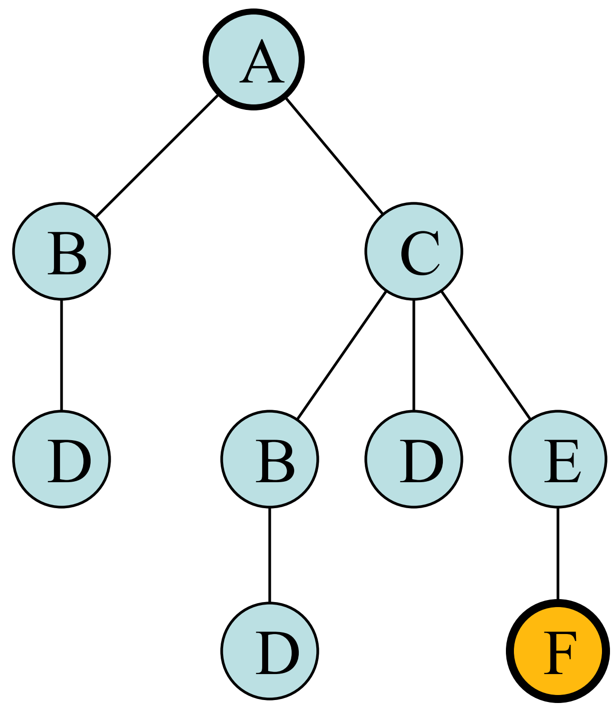

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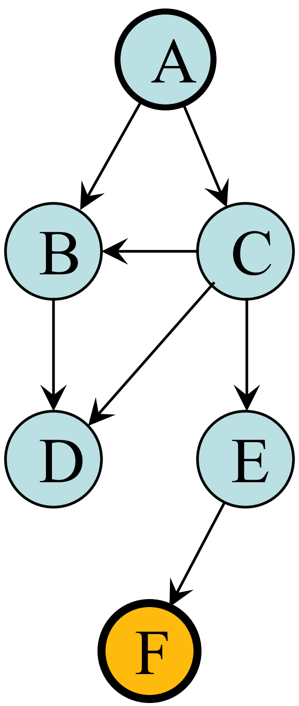

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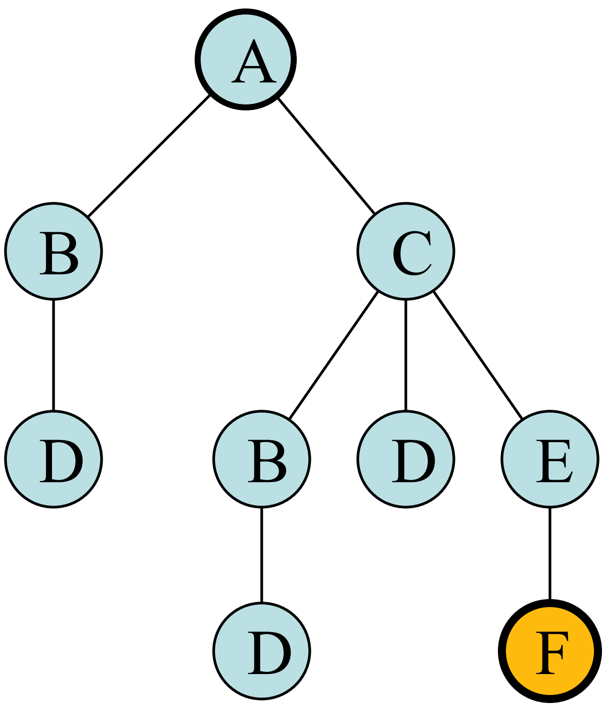

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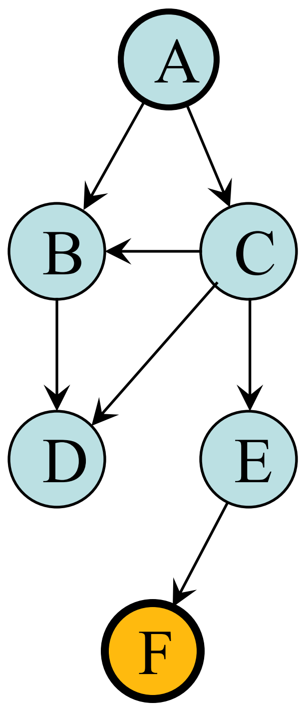
Example

State space graph



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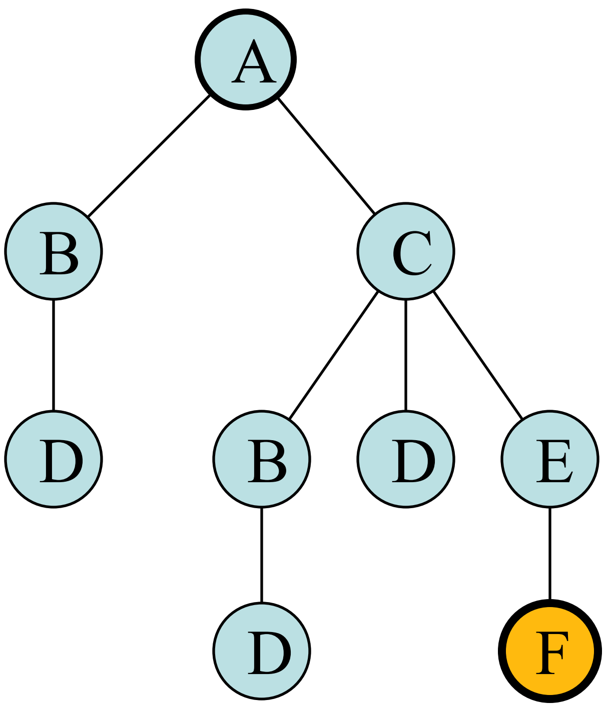

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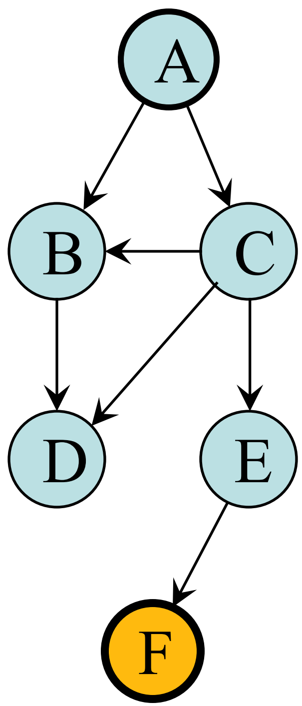
(E)

Example

State space graph



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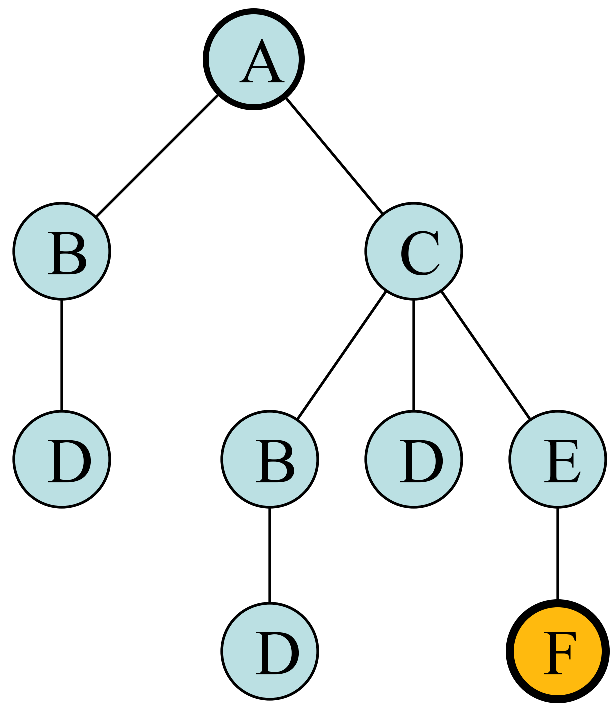

Queue

(A)

(B C)

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(B D E)

(D D E)

(D E)

(E)

(F)

Depth-First Search

- Complete?

- Optimal?

- Time complexity?

- Space complexity?

m = maximum depth of the search tree
(may be infinite)

Depth-First Search

- Complete? No
- Optimal?
- Time complexity?
- Space complexity?

m = maximum depth of the search tree
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Depth-First Search

- Complete? No
- Optimal? No
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m = maximum depth of the search tree
(may be infinite)

Depth-First Search

- Complete? No
- Optimal? No
- Time complexity? Exponential: $\mathbf{O(}\ \boldsymbol{b^m}\ \mathbf{)}$
- Space complexity?

m = maximum depth of the search tree
(may be infinite)

Depth-First Search

- Complete? No
- Optimal? No
- Time complexity? Exponential: **O(b^m)**
- Space complexity? Polynomial: **O(bm)**

m = maximum depth of the search tree (may be infinite)

What is the difference between the BFS / DFS that you learned from the algorithm / data structure course?

- Nothing, except:
  - Now you are applying them to solve an AI problem
  - The graph can be infinitely large
  - The graph does not need to be known ahead of time (you only need local information: Goal-state checker, Successor function)

Next lecture

- Informed search

- Start game solving / minimax search

- You should:
  - Read Chapter 3 of AIMA textbook
  - Start working on HW2

--------------------Supplementary slide---------------

- More examples
- More quiz questions

Example: MU-Puzzle

- States: Strings comprising the letters M, I, and U
- Initial state: MI
- Goal state: MU
- Operators: (where x stands for any string, *including the null string*)
  1. $x\,\mathrm{I} \rightarrow x\,\mathrm{IU}$ "Append U"
  2. $\mathrm{M}\,x \rightarrow \mathrm{M}\,x\,x$ "Replicate x"
  3. $x\mathrm{I\,I\,I}y \rightarrow x\mathrm{U}y$ "Replace III with U"
  4. $x\mathrm{UU}y \rightarrow xy$ "Drop UU"

- Path cost: one per step
- Try it
  – Can you draw the state-space diagram?
  – Are you guaranteed a solution?

M I
→ M I I
→ M I I I I
→ M U I
→ M U I U
→ M U I U U I U
→ M U I U U I U U I U U I U
→ …