Artificial Intelligence

CS 165A

Oct 29, 2020

Instructor: Prof. Yu-Xiang Wang

Today

→ Examples of heuristics in A*-search

→ Games and Adversarial Search

Recap: Search algorithms

- State-space diagram vs Search Tree
- Uninformed Search algorithms
  - BFS / DFS
  - Depth Limited Search
  - Iterative Deepening Search.
  - Uniform cost search.
- Informed Search (with an heuristic function h):
  - Greedy Best-First-Search. (not complete / optimal)
  - A* Search (complete / optimal if h is admissible)

Recap: Summary table of uninformed search

Criteria	BFS	Uniform-cost	DFS	Depth-limited	IDS	Bidirectional
Complete?	Yes$^{\#}$	Yes$^{\#\&}$	No	No	Yes$^{\#}$	Yes$^{\#+}$
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/e \rfloor})$	$O(b^m)$	$O(b^l)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/e \rfloor})$	$O(bm)$	$O(bl)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes$^{\$}$	Yes	No	No	Yes$^{\$}$	Yes$^{\$+}$

b: Branching factor
d: Depth of the shallowest goal
l: Depth limit
m: Maximum depth of search tree
e: The lower bound of the step cost

(Section 3.4.7 in the AIMA book.)

$^{\#}$: Complete if b is finite
$^{\&}$: Complete if step cost >= e
$^{\$}$: Optimal if all step costs are identical
$^{+}$: If both direction use BFS

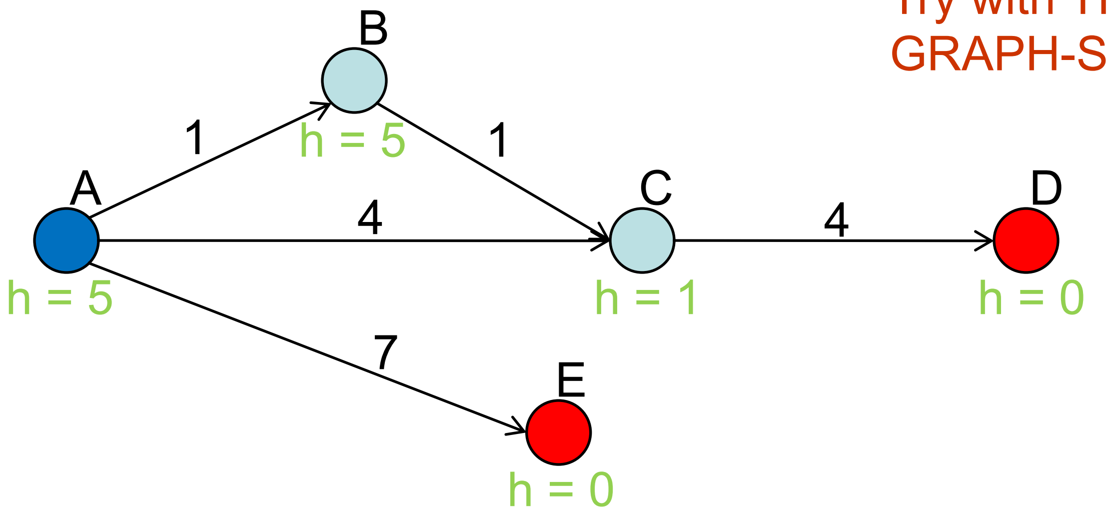
Recap: A* Search (Pronounced "A-Star")

- Uniform-cost search minimizes ***g(n)*** ("past" cost)
- Greedy search minimizes ***h(n)*** ("expected" or "future" cost)
- **"A*** Search" combines the two:
  - Minimize $f(n) = g(n) + h(n)$
  - Accounts for the "past" and the "future"
  - Estimates the cheapest solution (complete path) through node ***n***

function A*-SEARCH(*problem*, ***h***) **returns** a solution or failure
  return BEST-FIRST-SEARCH(*problem*, ***f***)

Recap: Avoiding Repeated States using A* Search

- Is GRAPH-SEARCH optimal with A*?



Try with TREE-SEARCH and GRAPH-SEARCH

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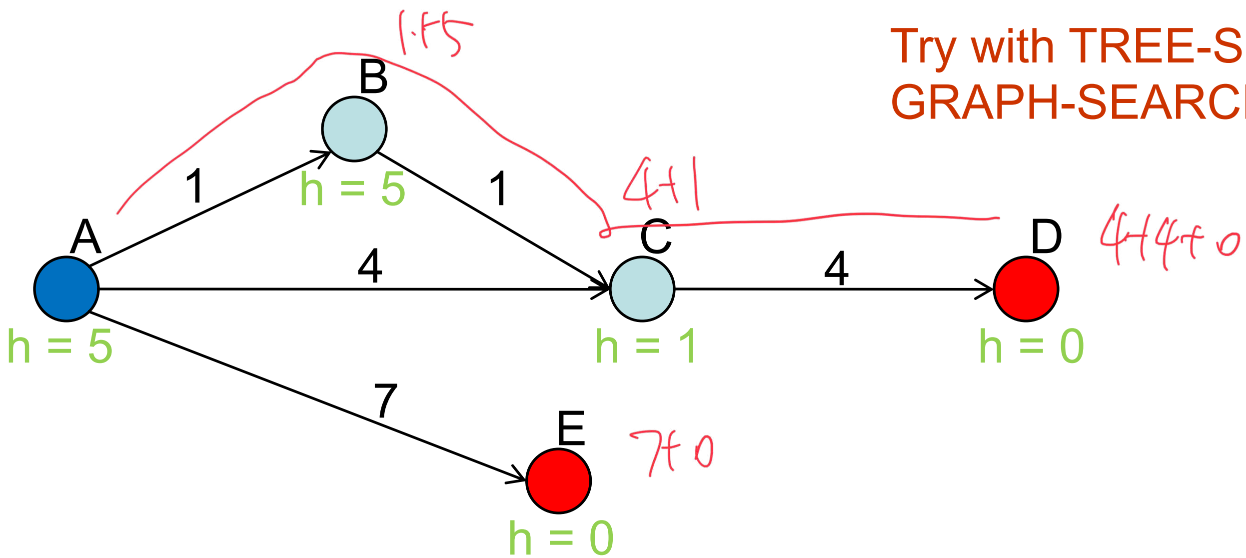
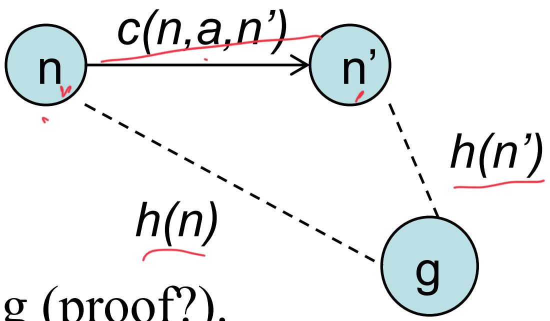


Try with TREE-SEARCH and GRAPH-SEARCH

Graph Search
Step 1: Among B, C, E, Choose C
Step 2: Among B, E, D, Choose B
Step 3: Among D, E, Choose E. (you are not going to select C again) 8 87

Recap: Consistency (Monotonicity) of heuristic h

- A heuristic is consistent (or monotonic) provided
  - for any node n, for any successor n' generated by action a with cost c(n,a,n')
    - $h(n) \leq c(n,a,n') + h(n')$
  - akin to triangle inequality.
  - guarantees admissibility (proof?).
  - values of f(n) along any path are non-decreasing (proof?).
    - Contours of constant f in the state space
- GRAPH-SEARCH using consistent f(n) is optimal.
- Note that h(n) = 0 is consistent and admissible.



This lecture

- Example of heuristics / A* search
  - Effective branching factor

- Games

- Adversarial Search

Heuristics

- What's a heuristic for
  - Driving distance (or time) from city A to city B ? Straight line dist.
  - 8-puzzle problem ? # of misplaced tiles
  - M&C ? # of ppl on the LHS
  - Robot navigation ? # of dirty tiles

Heuristics

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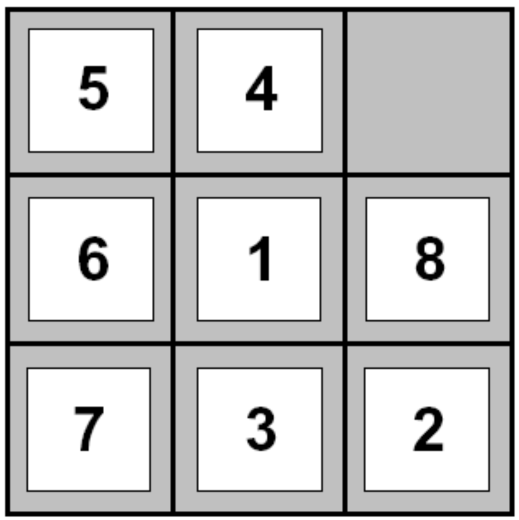
$h(n) - h(n') \leq |h(n) - h(n')| \leq 1$

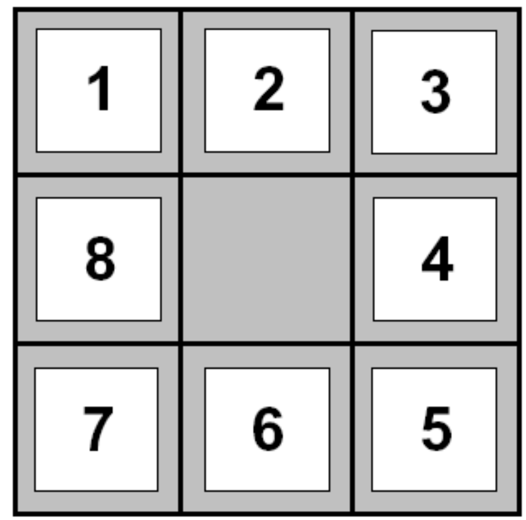
$\therefore h(n) \leq 1 + h(n')$

- Are the above heuristics admissible? Consistent?

Example: 8-Puzzle



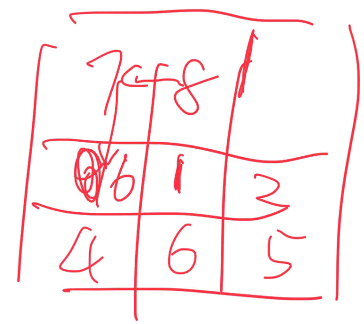
Start State

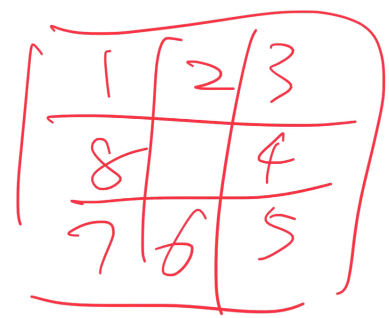


Goal State

Comparing and combining heuristics

- Heuristics generated by considering relaxed versions of a problem.
- Heuristic h_1 for 8-puzzle
  - Number of out-of-order tiles
- Heuristic h_2 for 8-puzzle
  - Sum of Manhattan distances of each tile
- h_2 dominates h_1 provided $h_2(n) \geq h_1(n)$.
  - h_2 will likely prune more than h_1.
- $h =$ $\max(h_1, h_2, .., h_n)$ is
  - admissible if each h_i is
  - consistent if each h_i is
- Cost of sub-problems and pattern databases
  - Cost for 4 specific tiles
  - Can these be added for disjoint sets of tiles?





Effective Branching Factor

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- Though <u>informed</u> search methods may have poor *worst-case* performance, they often do quite well if the heuristic is good
  - Even if there is a huge branching factor

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- One way to quantify the effectiveness of the heuristic: the **effective branching factor, b^***
  - N: total number of nodes expanded
  - d: solution depth
  - $N = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d$

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- For a good heuristic, b^* is close to 1

Example: 8-puzzle problem

Averaged over 100 trials each at different solution lengths

	Search Cost			Effective Branching Factor		
d	IDS	$A^*(h_1)$	$A^*(h_2)$	IDS	$A^*(h_1)$	$A^*(h_2)$
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

Ave. # of nodes expanded

Solution length

Memory Bounded Search

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- Solution: Memory-bounded A* search
  - Iterative Deepening A* (IDA*)
  - Simplified Memory-bounded A* (SMA*) Beam Search
  - (Read the textbook for more details.)

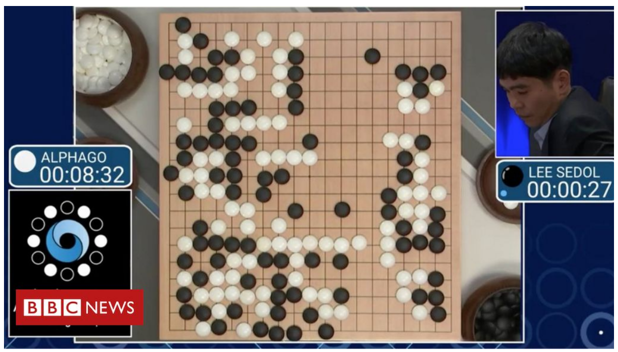
Summary of informed search

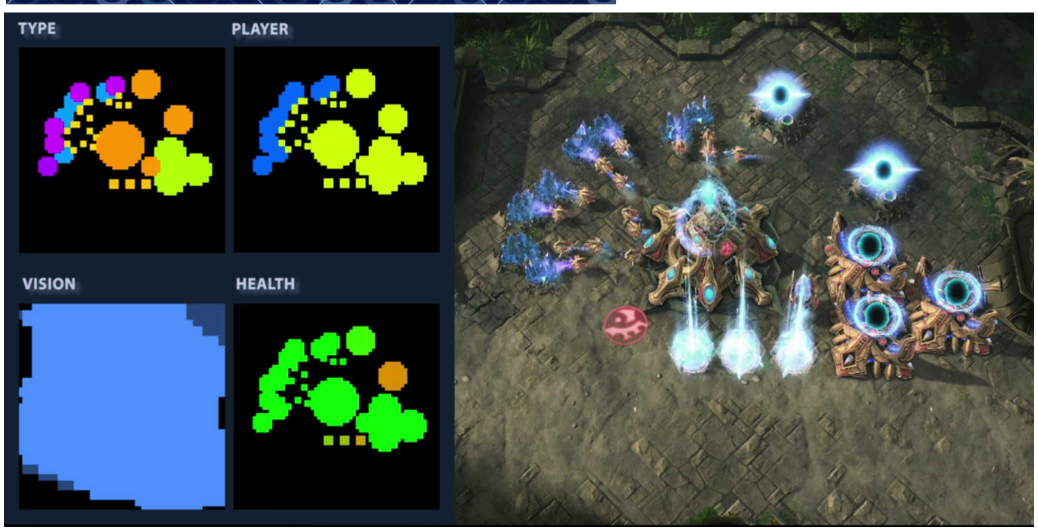
- How to use a heuristic function to improve search
  - Greedy Best-first search + Uniform-cost search = A* Search

- When is A* search optimal?
  - h is Admissible (optimistic) for Tree Search
  - h is Consistent for Graph Search

- Choosing heuristic functions
  - A good heuristic function can reduce time/space cost of search by orders of magnitude.
  - Good heuristic function may take longer to evaluate.

Games and Adversarial Search





- Games: problem setup
- Minimax search
- Alpha-beta pruning

Illustrative example of a simple game (1 min discussion)

Example: game 1

You choose one of the three bins.
I choose a number from that bin.
Your goal is to maximize the chosen number.

A		B		C	
-50	50	1	3	-5	15

$E[r]$
$= 15 \cdot 0.5$
$+ (-5) \cdot 0.5$

(Example taken from Liang and Sadigh)

Game as a search problem

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Game as a search problem

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- ACTIONS(s): Returns the legal moves.
- RESULT(s, a): Output the state we transition to.
- TERMINAL-TEST(s): Returns True if the game is over.
- UTILITY(s,p): The payoff of player p at terminal state s.

[illegible]

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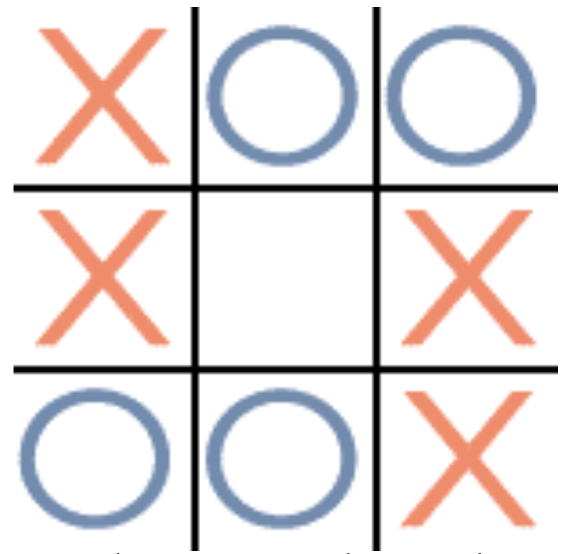
Two-player, Turn-based, Perfect information, Deterministic, Zero-Sum Game

- Two-player: Tic-Tac-Toe, Chess, Go!
- Turn-based: The players take turns in round-robin fashion.
- Perfect information: The State is known to everyone
- Deterministic: Nothing is random
- Zero-sum: The total payoff for all players is a constant.
  - *The 8-puzzle is a one-player, perfect info, deterministic, zero-sum game.*
  - *How about Rock-Paper-Scissors?*
  - *How about Monopoly?*
  - *How about Starcraft?*

Tic-Tac-Toe



- The first player is **X** and the second is **O**
- Object of game: get three of your symbol in a horizontal, vertical or diagonal row on a 3x3 game board
- **X** always goes first
- Players alternate placing **X**s and **O**s on the game board
- Game ends when a player has three in a row (a wins) or all nine squares are filled (a draw)

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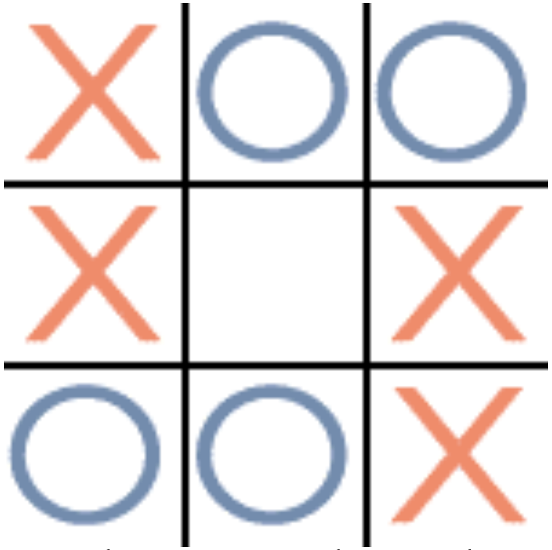

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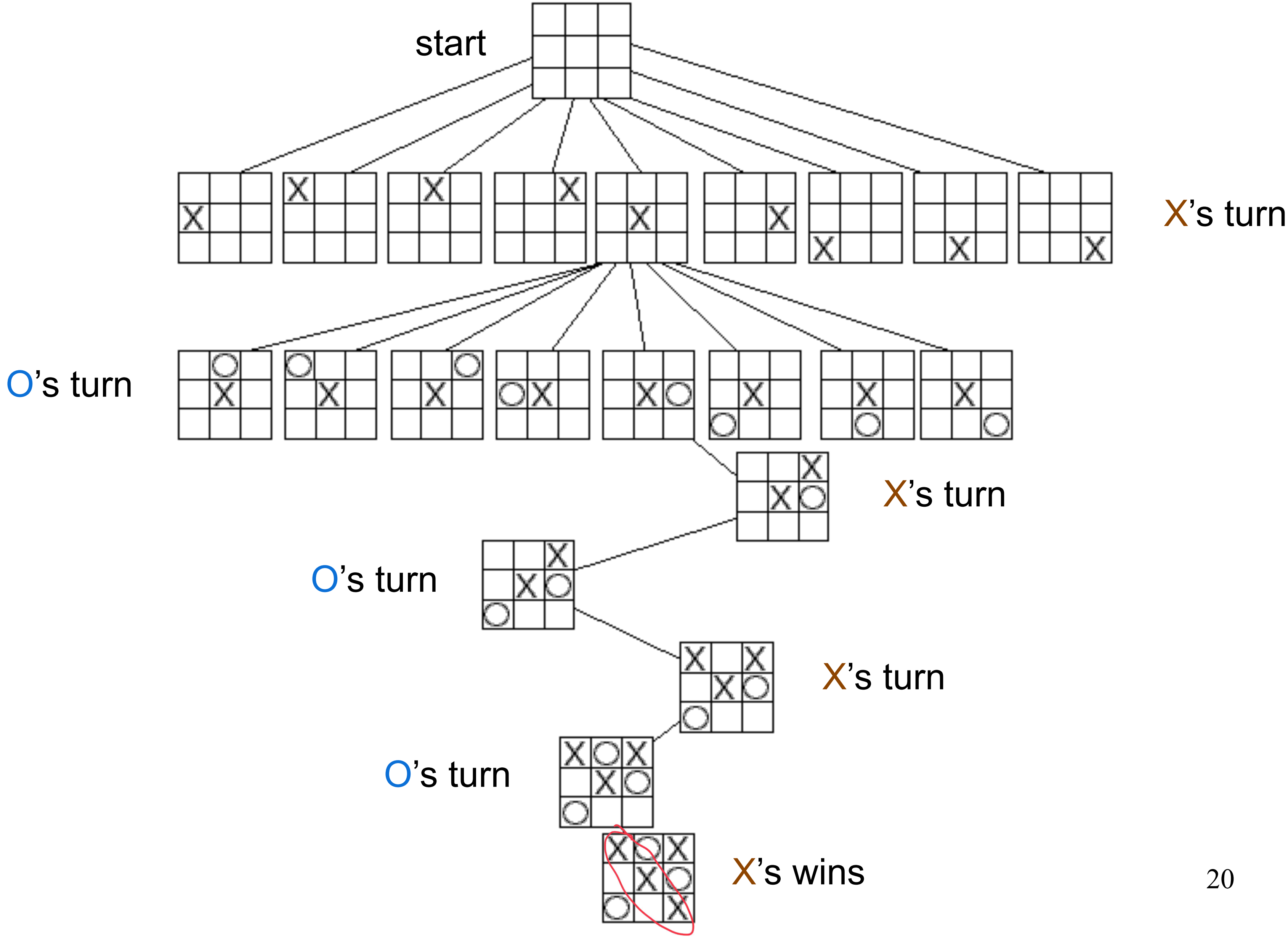
What's the state, action, transition, payoff for Tic-Tac-Toe?

Partial game tree for Tic-Tac-Toe



Game trees

- A game tree is like a search tree in many ways …
  - nodes are search states, with full details about a position
    - characterize the arrangement of game pieces on the game board
  - edges between nodes correspond to moves
  - leaf nodes correspond to a set of goals
    - { win, lose, draw }
    - usually determined by a score for or against player
  - at each node it is one or other player's turn to move
- A game tree is not like a search tree because you have an opponent!

Two players: MIN and MAX

- In a zero-sum game:
  - payoff to Player 1 = - payoff to Player 2

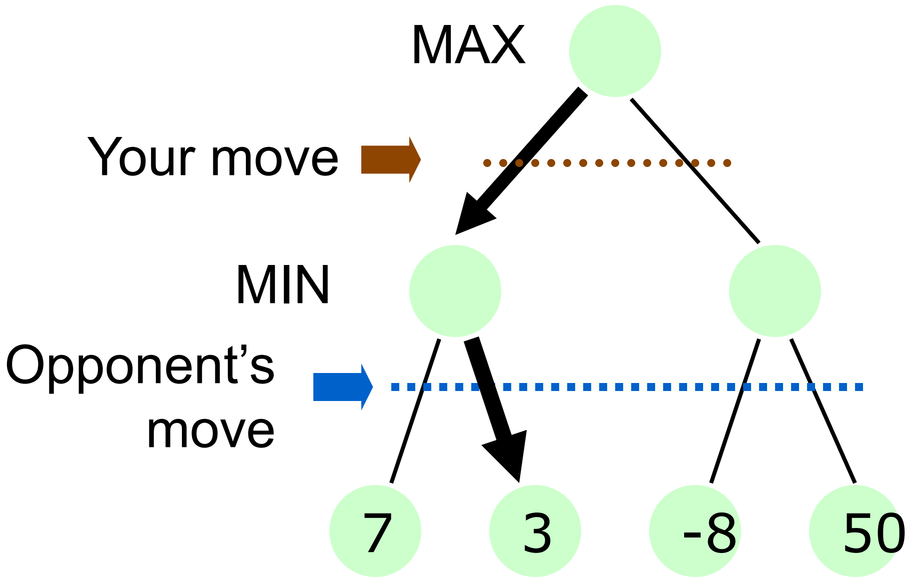
- The goal of Player 1 is to maximizing her payoff.

- The goal of Player 2 is to maximizing her payoff as well
  - Equivalent to minimizing Player 1's payoff.

Minimax search

- Assume that both players play perfectly
  - do not assume player will miss good moves or make mistakes
- Score(s): The score that MAX will get towards the end if both player play perfectly from s onwards.
- Consider MIN's strategy
  - MIN's best strategy:
    - choose the move that minimizes the score that will result when MAX chooses the maximizing move
  - MAX does the opposite

Minimaxing



- Your opponent will choose smaller numbers
- If you move left, your opponent will choose 3
- If you move right, your opponent will choose -8
- Thus your choices are only 3 or -8
- You should move left

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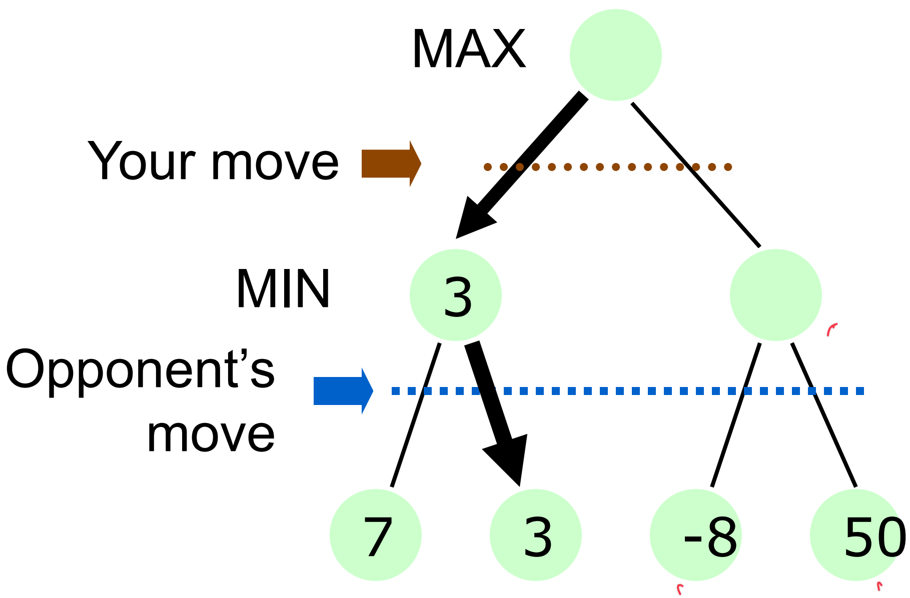

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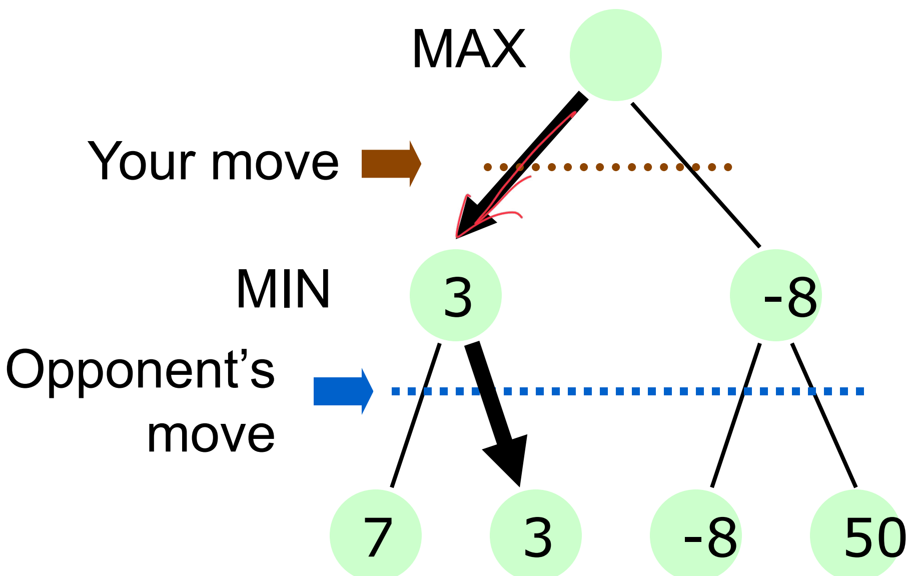

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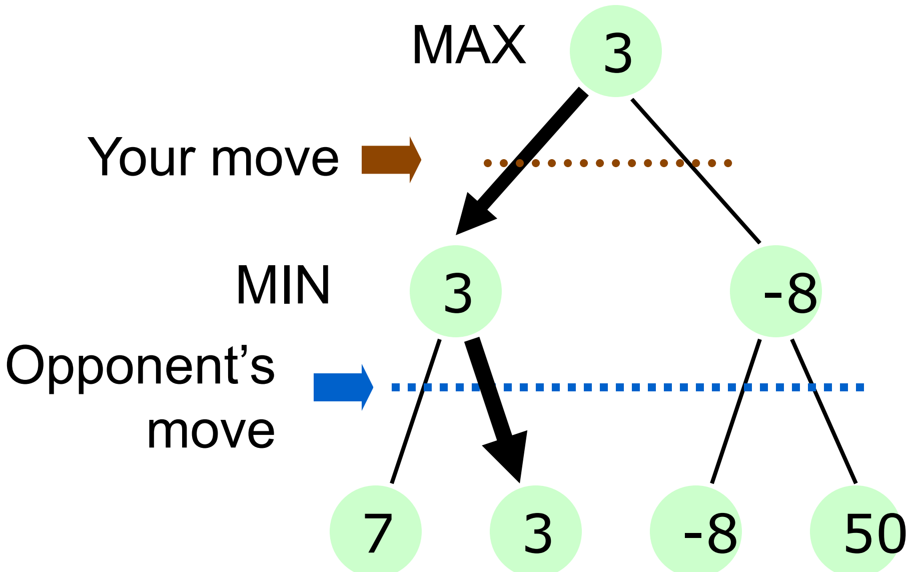

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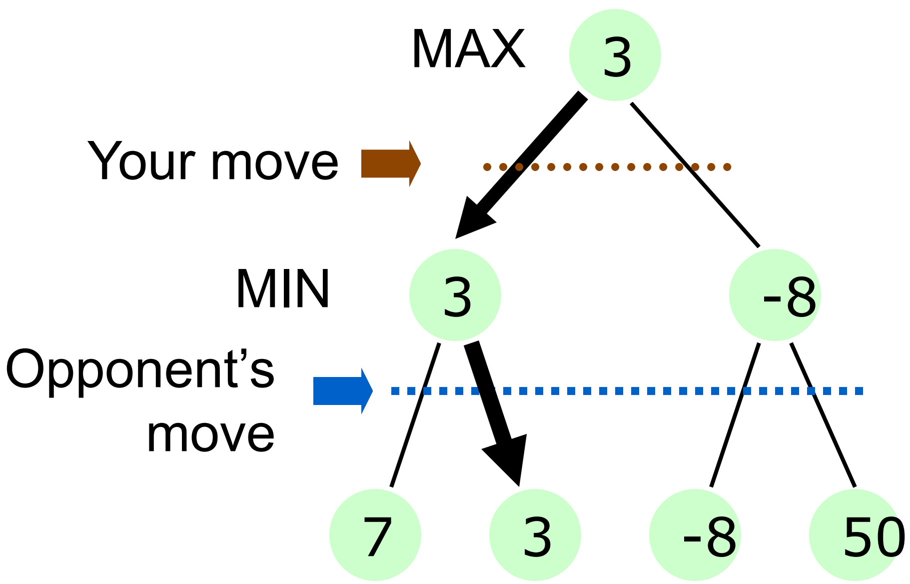

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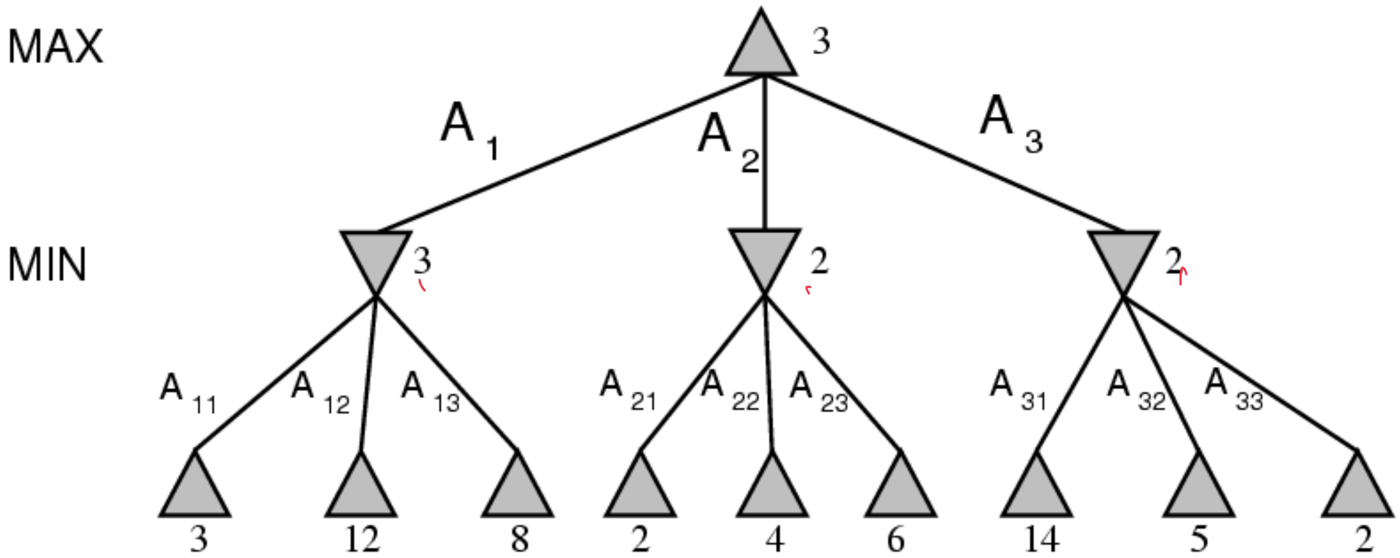
Each move is called a "ply". One round is K-plies for a K-player game.

Minimax example

Which move to choose?



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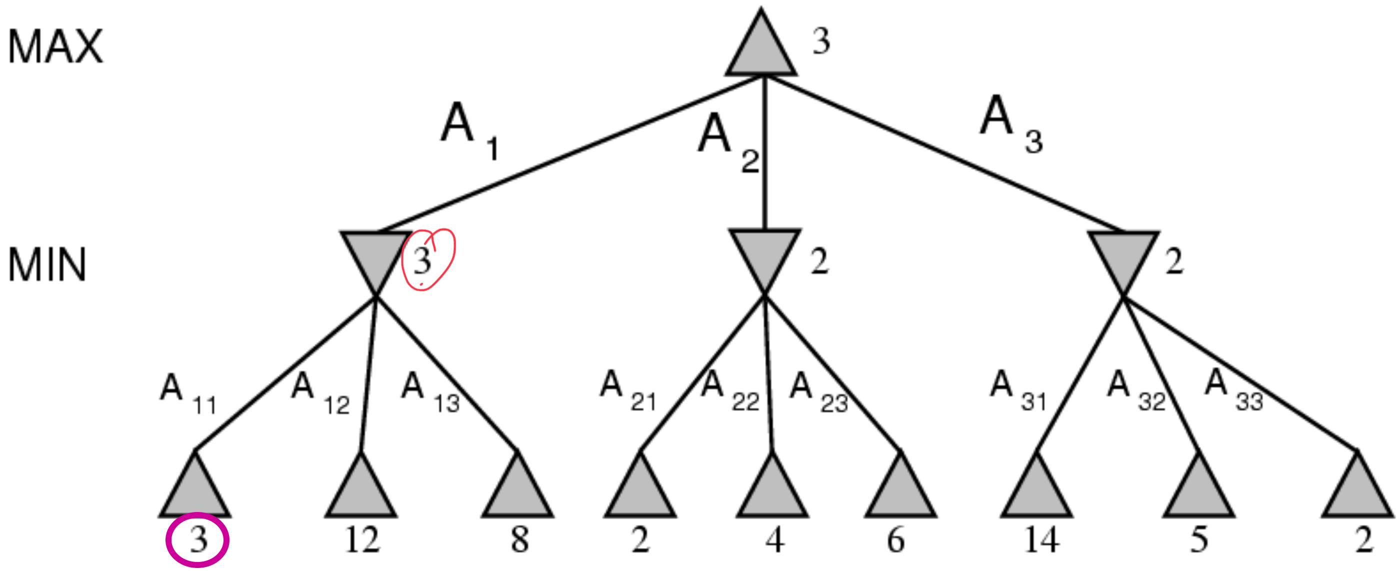

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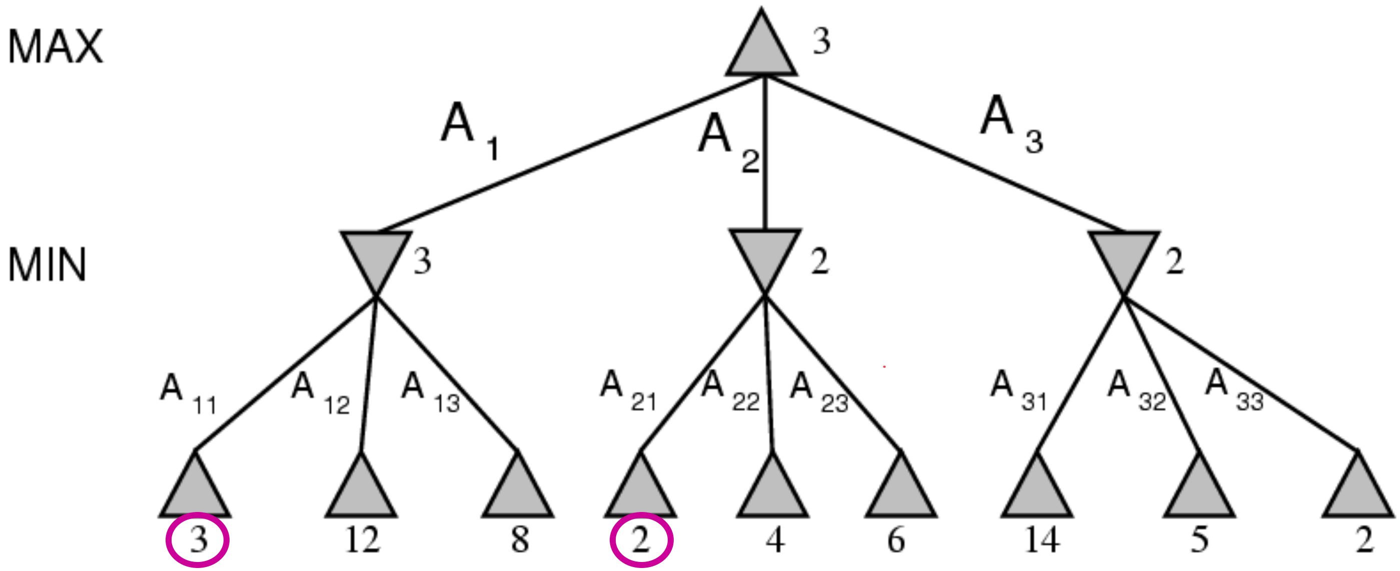

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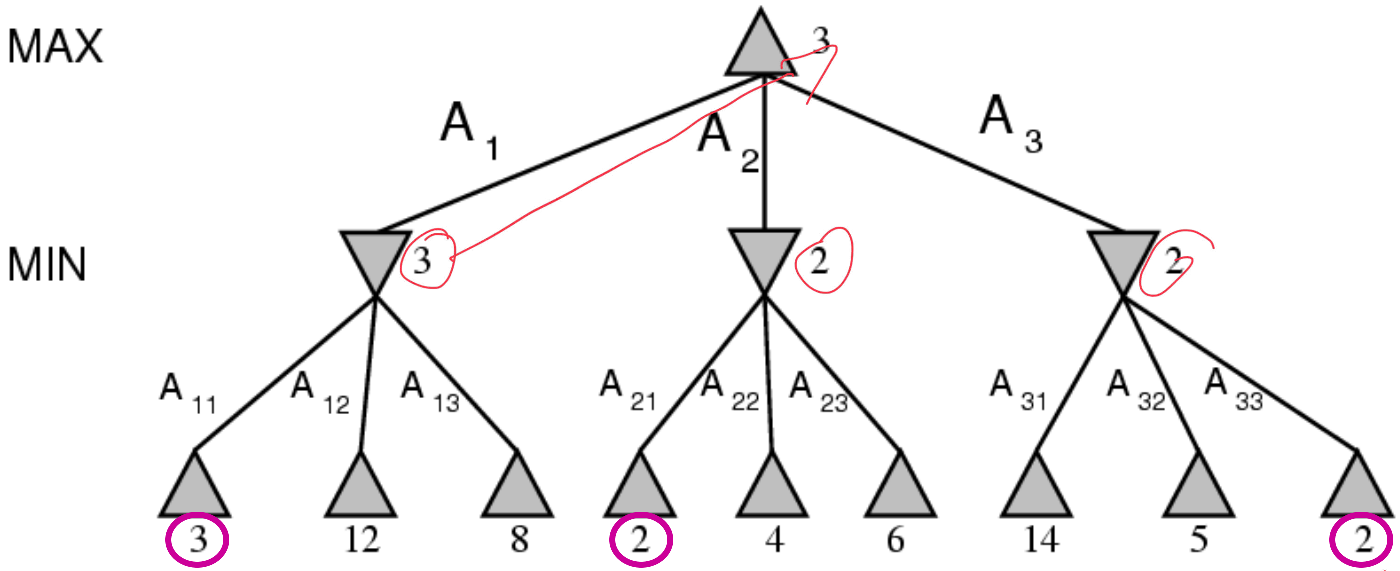

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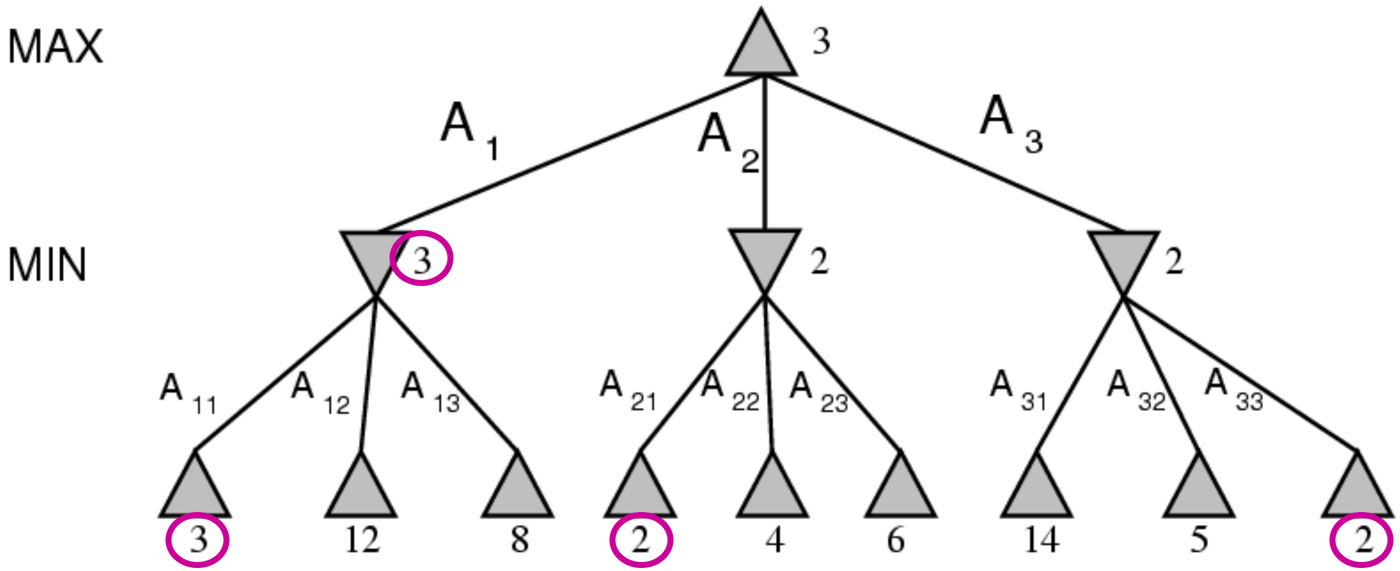

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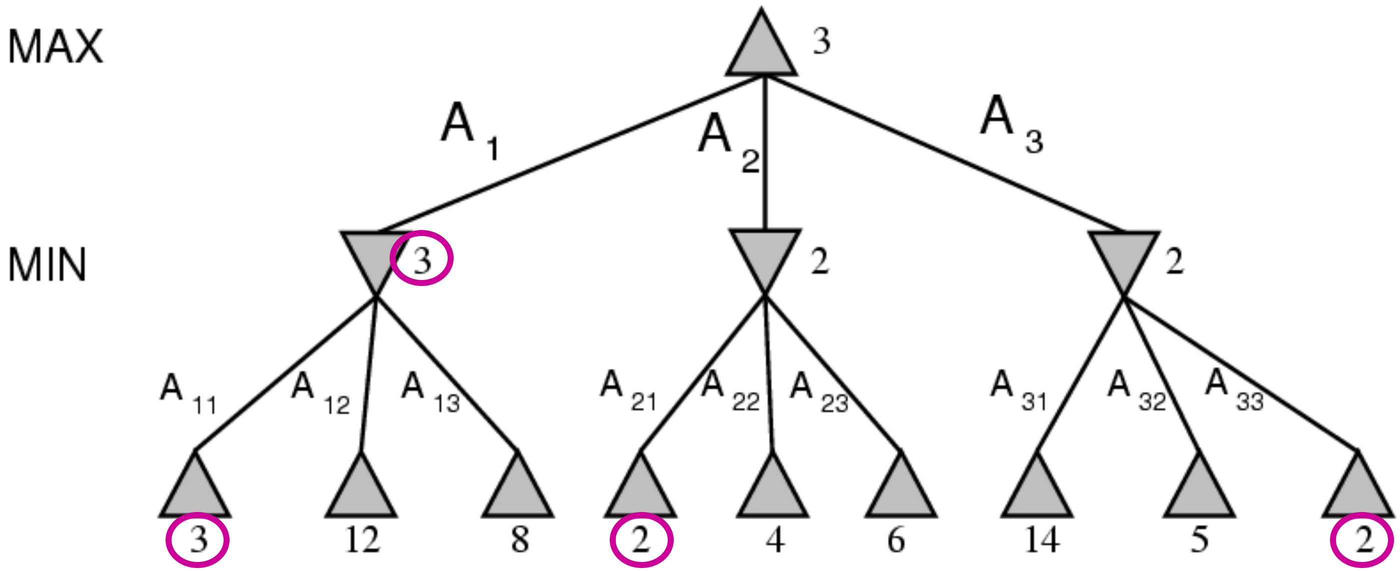
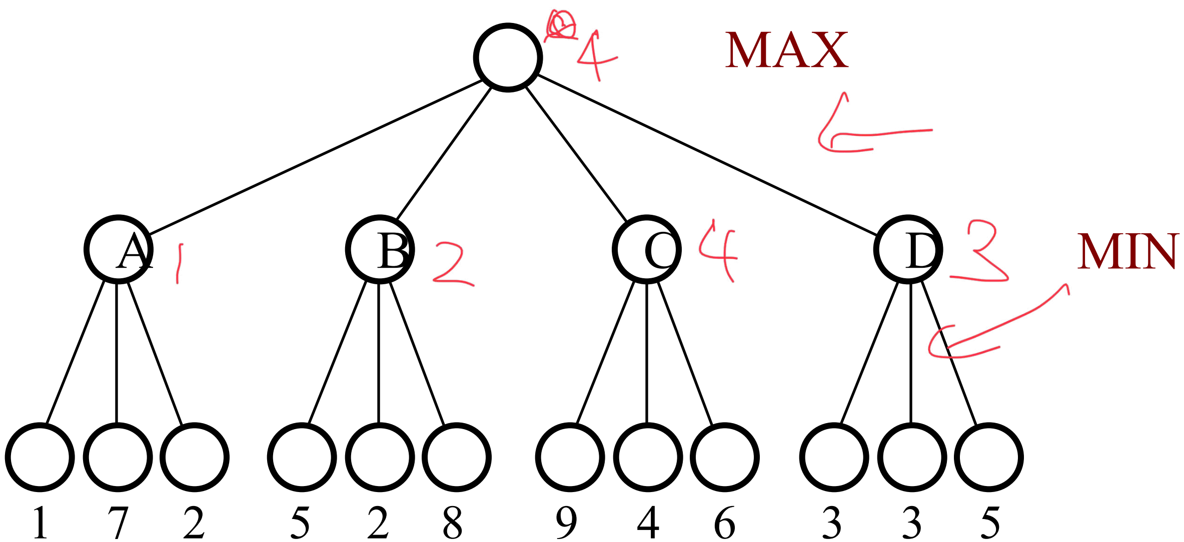


The **minimax decision** is move $\mathbf{A_1}$

Another example

- In the game, it's your move. Which move will the minimax algorithm choose – A, B, C, or D? What is the minimax value of the root node and nodes A, B, C, and D?



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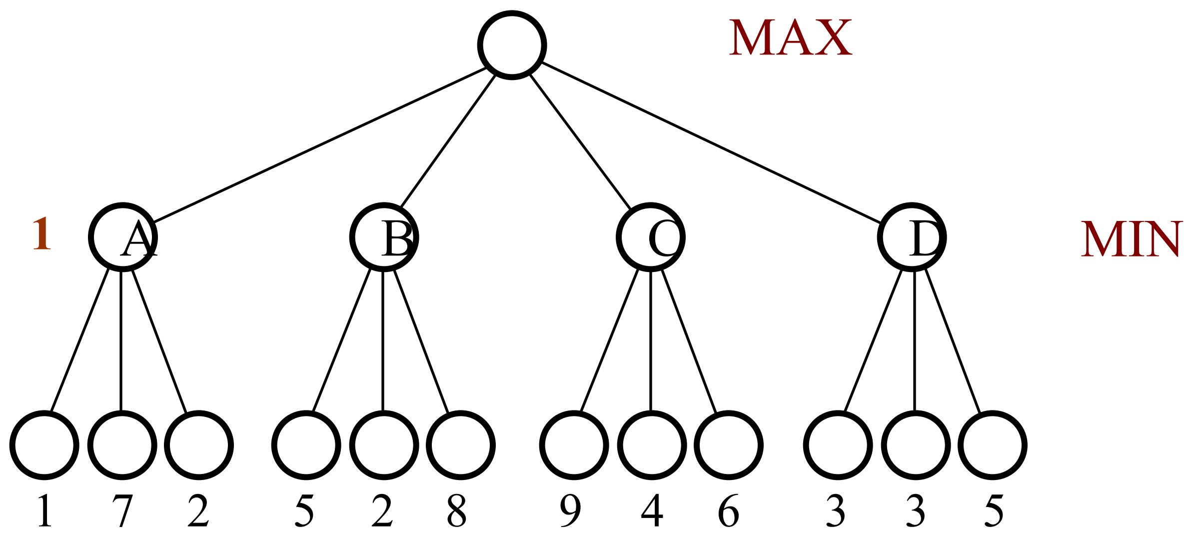

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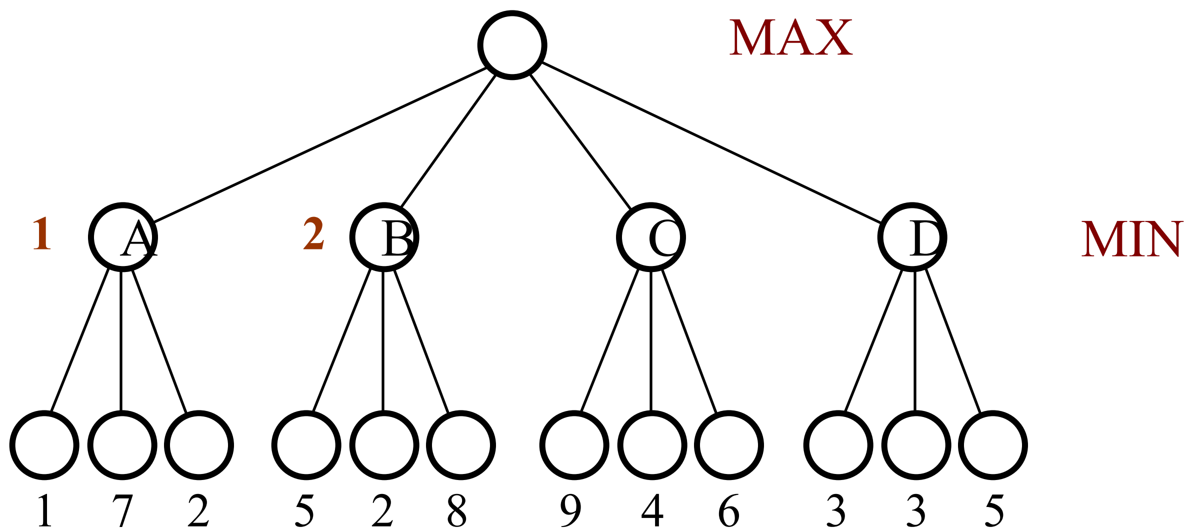

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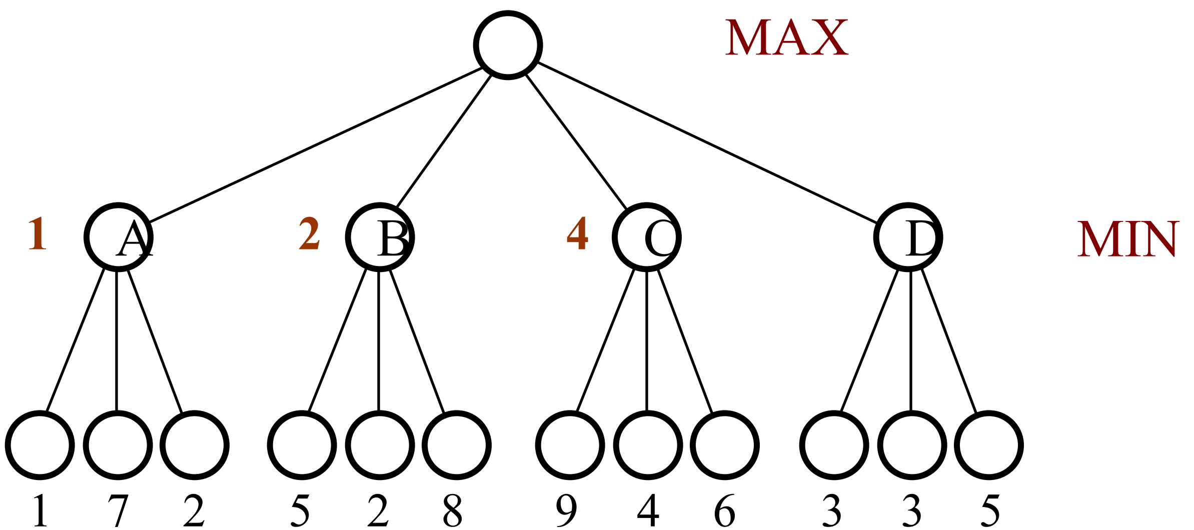

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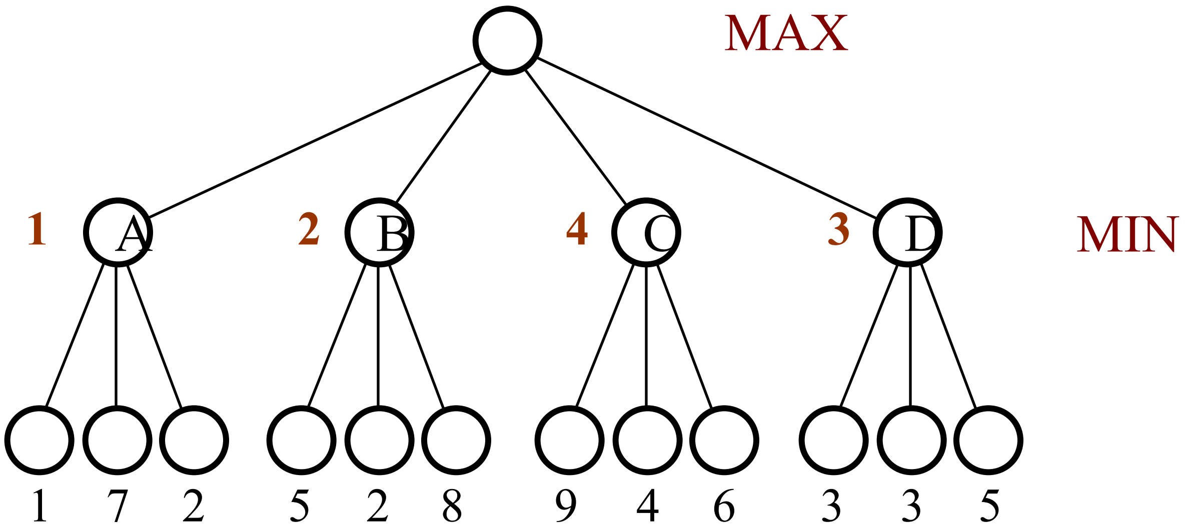

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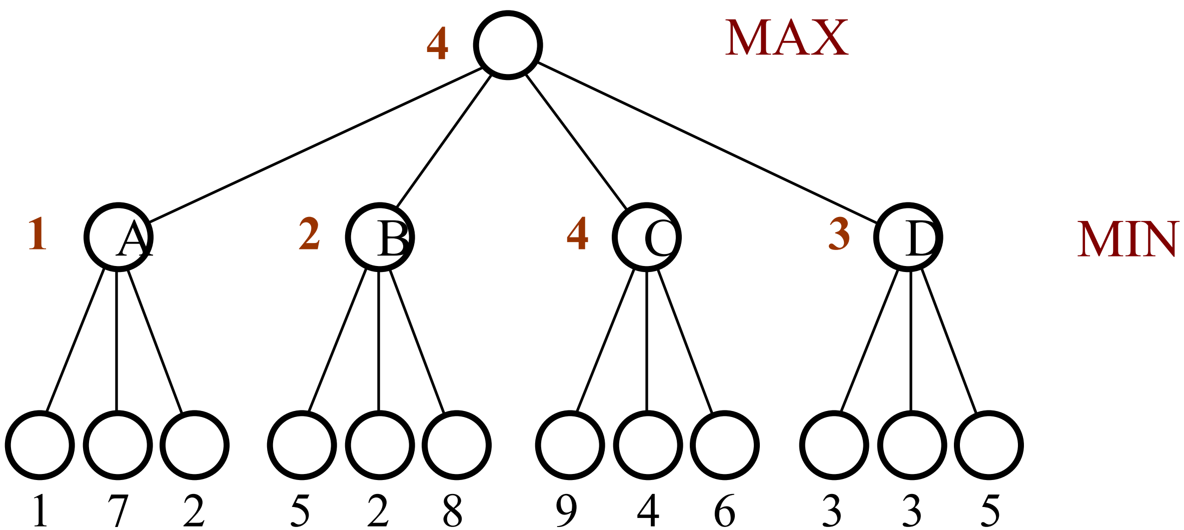

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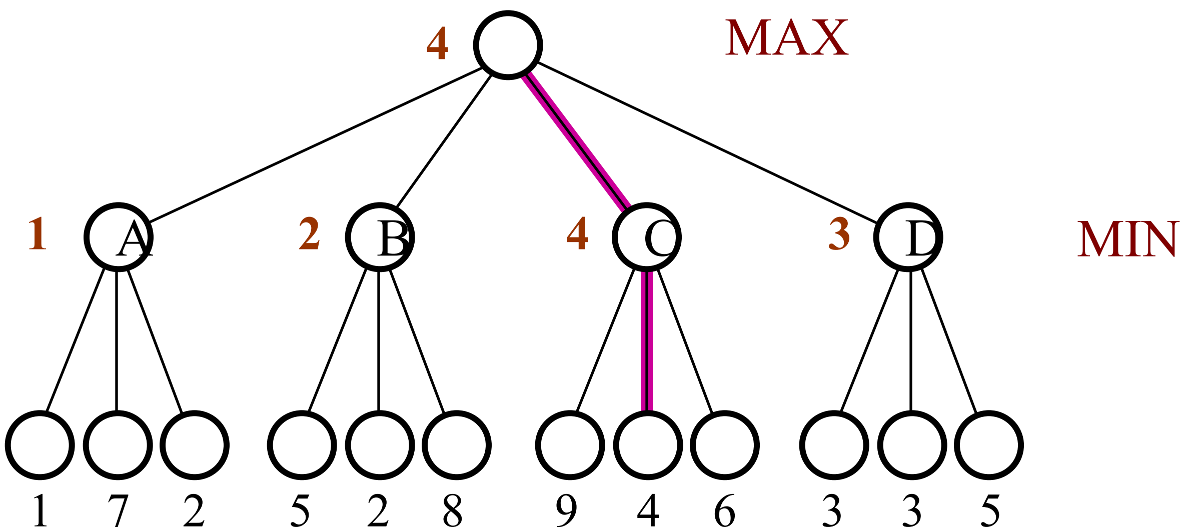

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- Depth-first search is used to implement minimax
  - Expand all the way down to leaf nodes
  - Recursive implementation

Minimax properties

- Optimal?

  Yes, against an optimal opponent, **if** the tree is finite

- Complete?

  Yes, **if** the tree is finite

- Time complexity?

  Exponential: **O(** $\boldsymbol{b^m}$ **)**

- Space complexity?

  Polynomial: **O(** $\boldsymbol{bm}$ **)**

But this could take forever…

- Exact search is intractable
  - Tic-Tac-Toe is 9! = 362,880
  - For chess, $b \approx 35$ and $m \approx 100$ for "reasonable" games
  - Go is 361! $\approx 10^{750}$

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  - For chess, $b \approx 35$ and $m \approx 100$ for “reasonable” games
  - Go is 361! $\approx 10^{750}$

- Idea 1: Pruning

But this could take forever…

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- Idea 1: Pruning

- Idea 2: Cut off early and use a heuristic function

Pruning

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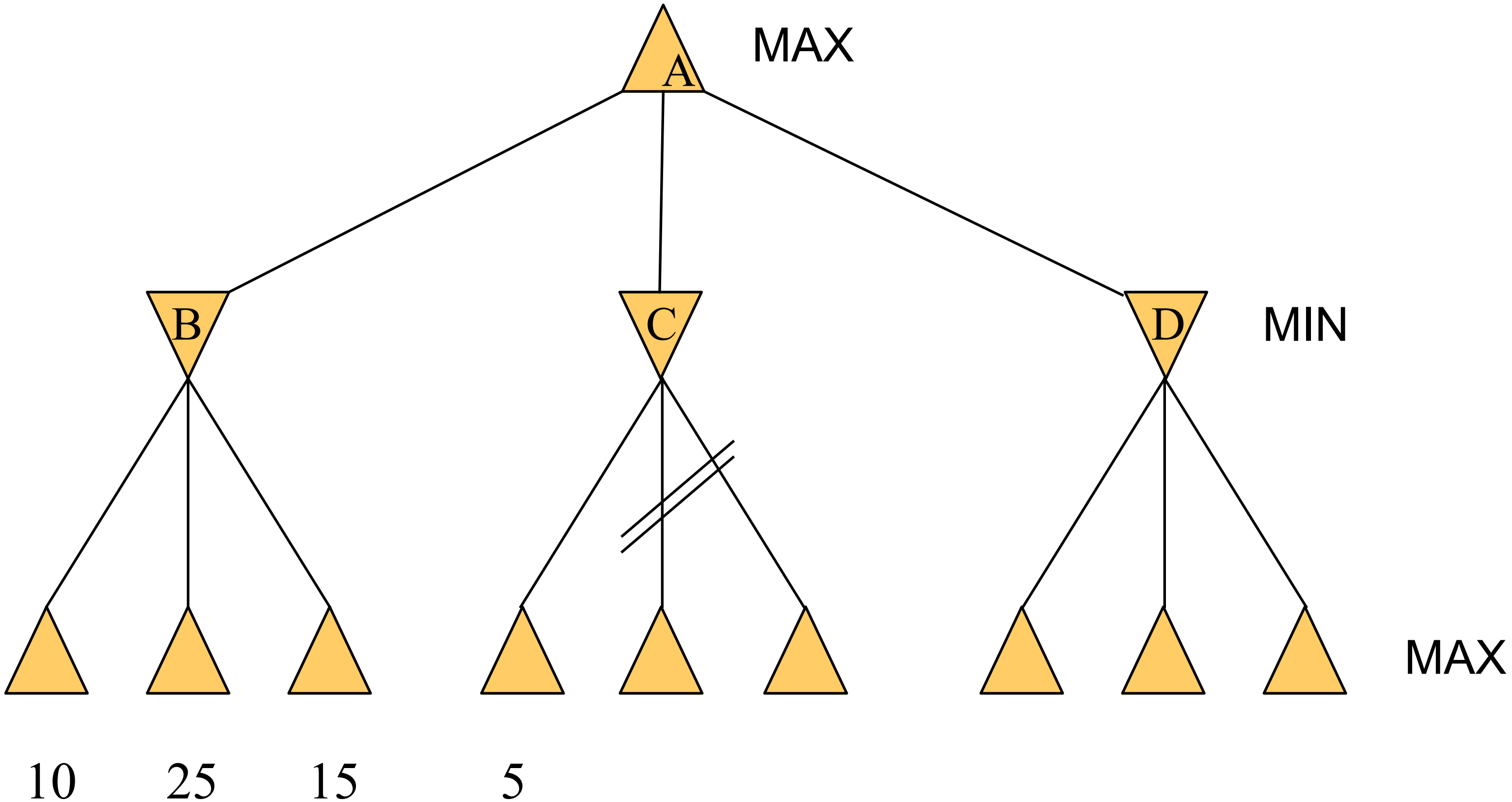
- **Pruning** – eliminating a branch of the search tree from consideration
- **Alpha-beta pruning**, applied to a minimax tree, returns the same "best" move, while pruning away unnecessary branches
  - Many fewer nodes might be expanded
  - Hence, smaller effective branching factor
  - …and deeper search
  - …and better performance
    - Remember, minimax is *depth-first* search

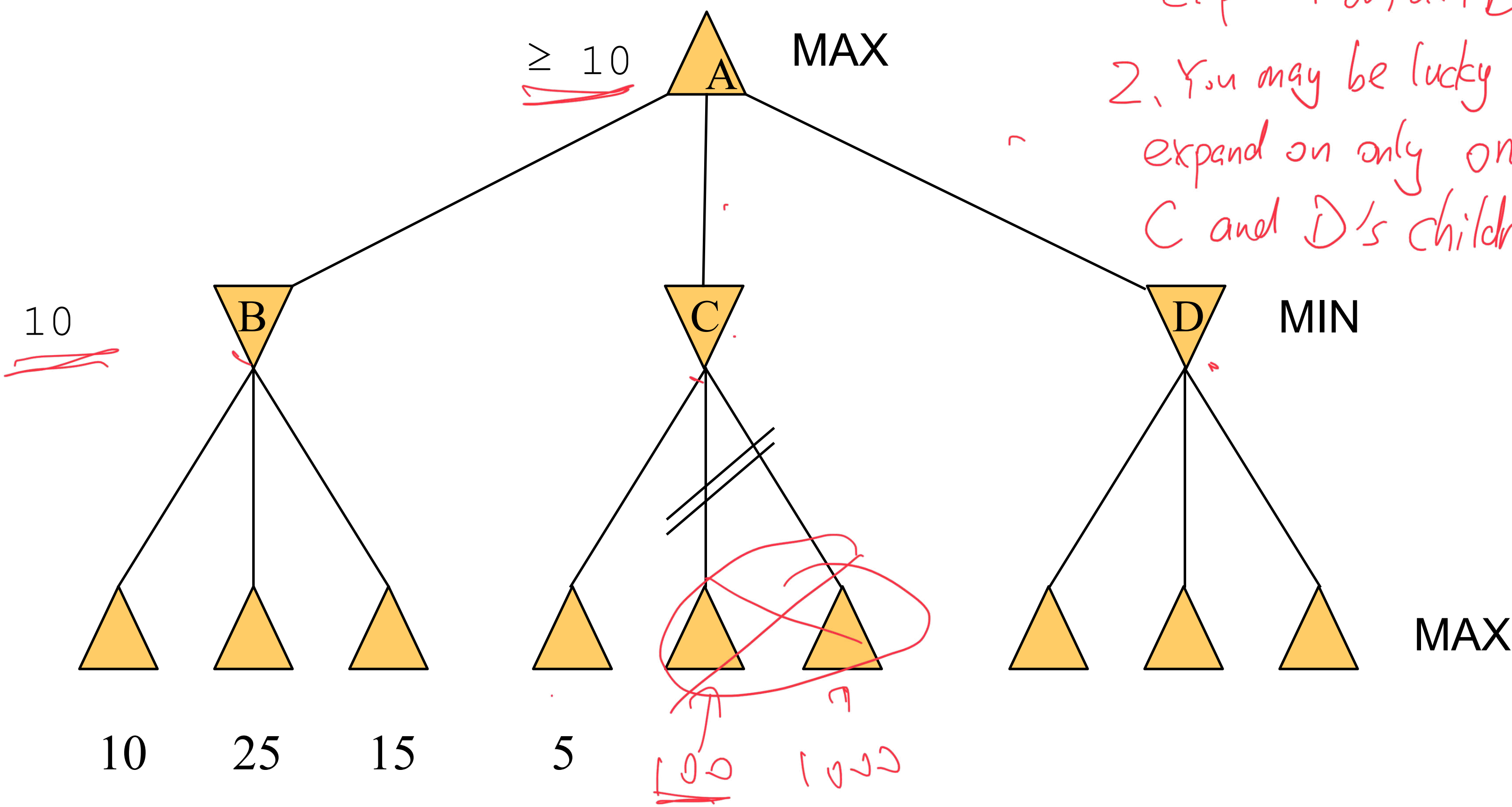
Alpha pruning



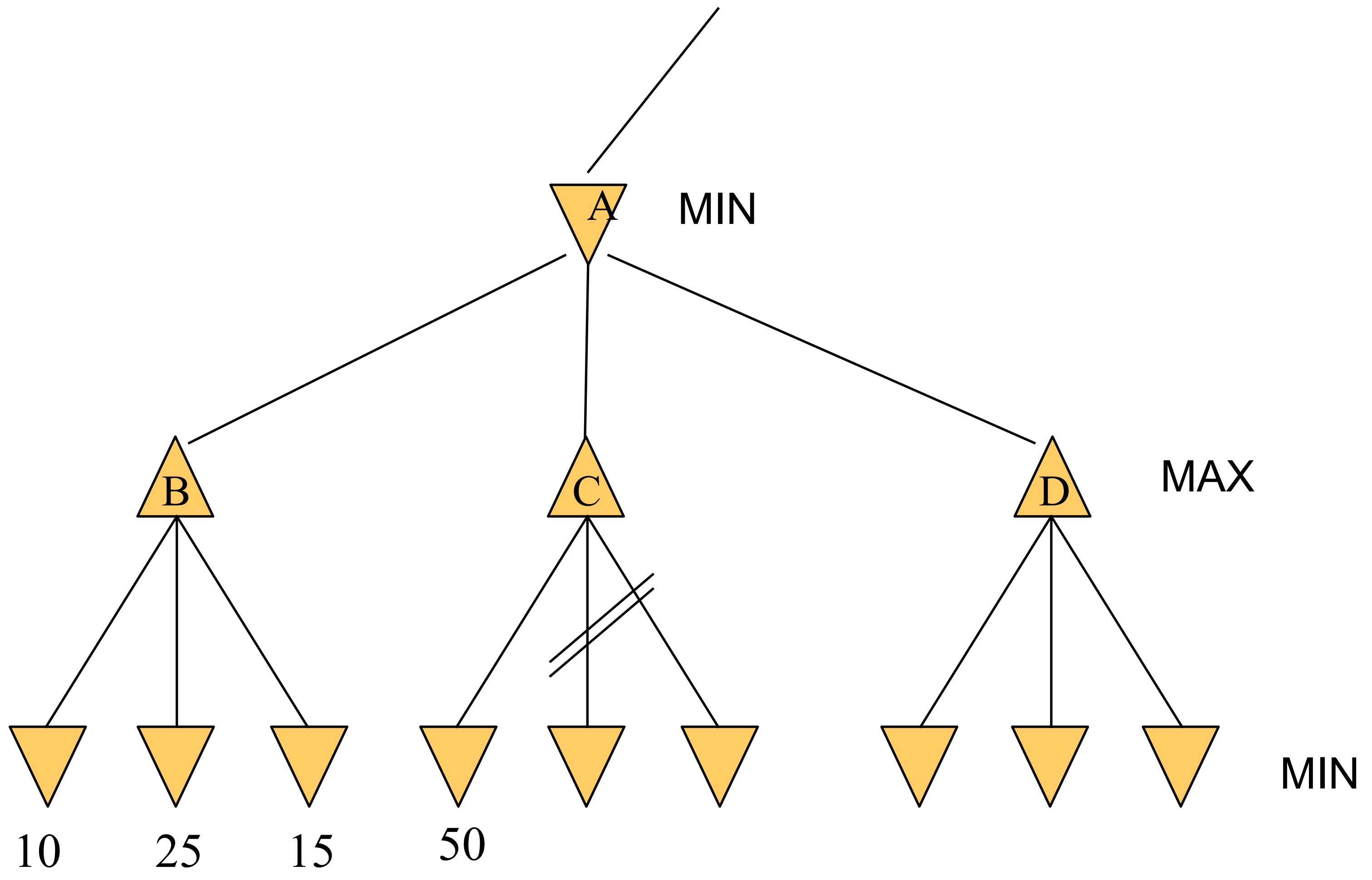
Alpha pruning

Notes:

1. You still have to expand on all B, C, D
2. You may be lucky and expand on only one of C and D's children.



Beta pruning



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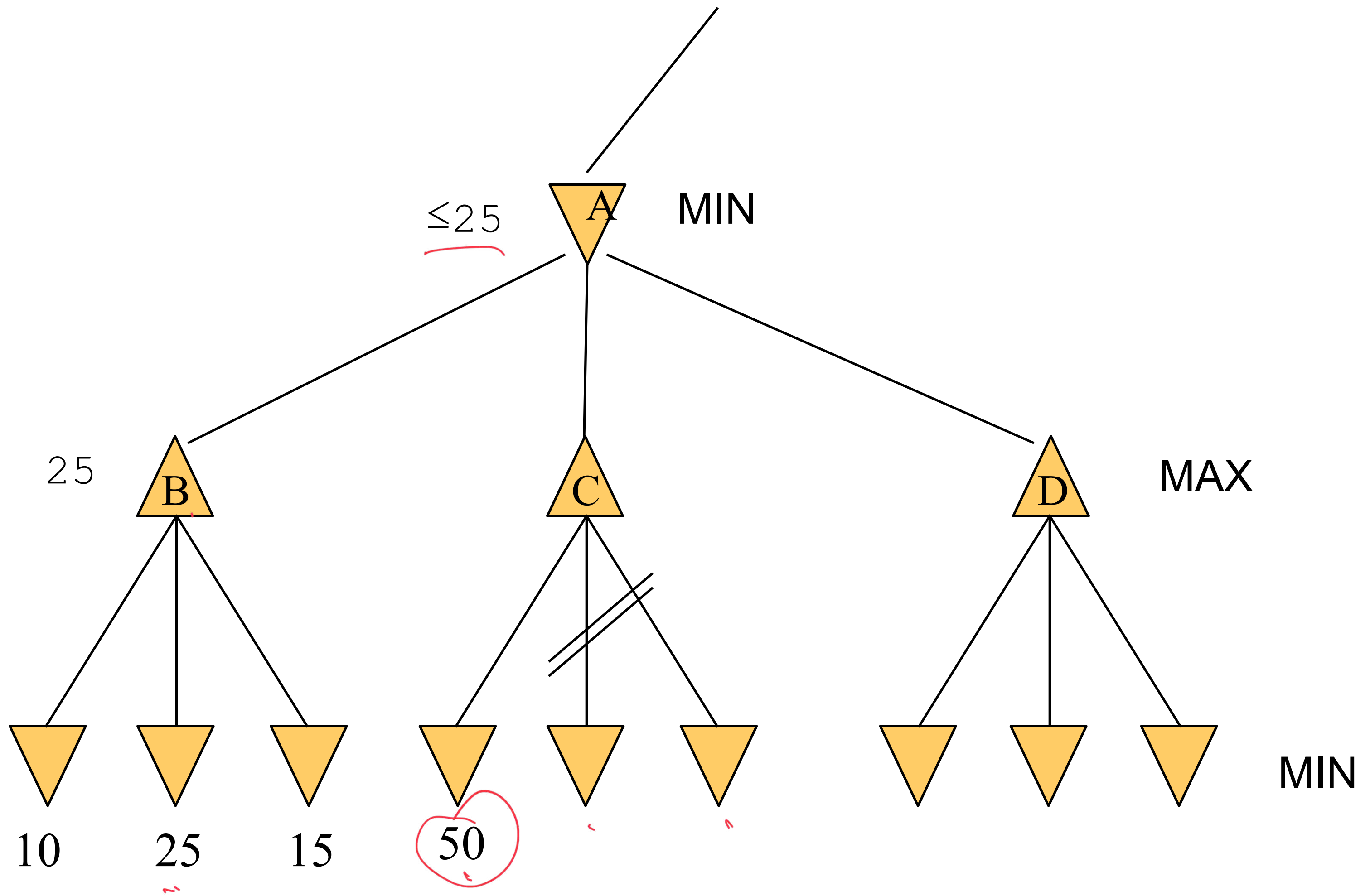

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- For specific games like Chess, you can get to almost perfect ordering.

Heuristic (Evaluation function)

- It is usually impossible to solve games completely
- Rather, cut the search off early and apply a heuristic evaluation function to the leaves
  - ***h*(*s*)** estimates the expected utility of the game from a given position (node/state) ***s***
  - like depth bounded depth first, lose completeness
  - Explore game tree using combination of evaluation function and search

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  - Explore game tree using combination of evaluation function and search
- The performance of a game-playing program depends on the quality (and speed!) of its evaluation function

Heuristics (Evaluation function)

- Typical evaluation function for game: weighted linear function
  - $h(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_d f_d(s)$
  - *weights* · *features* [dot product]

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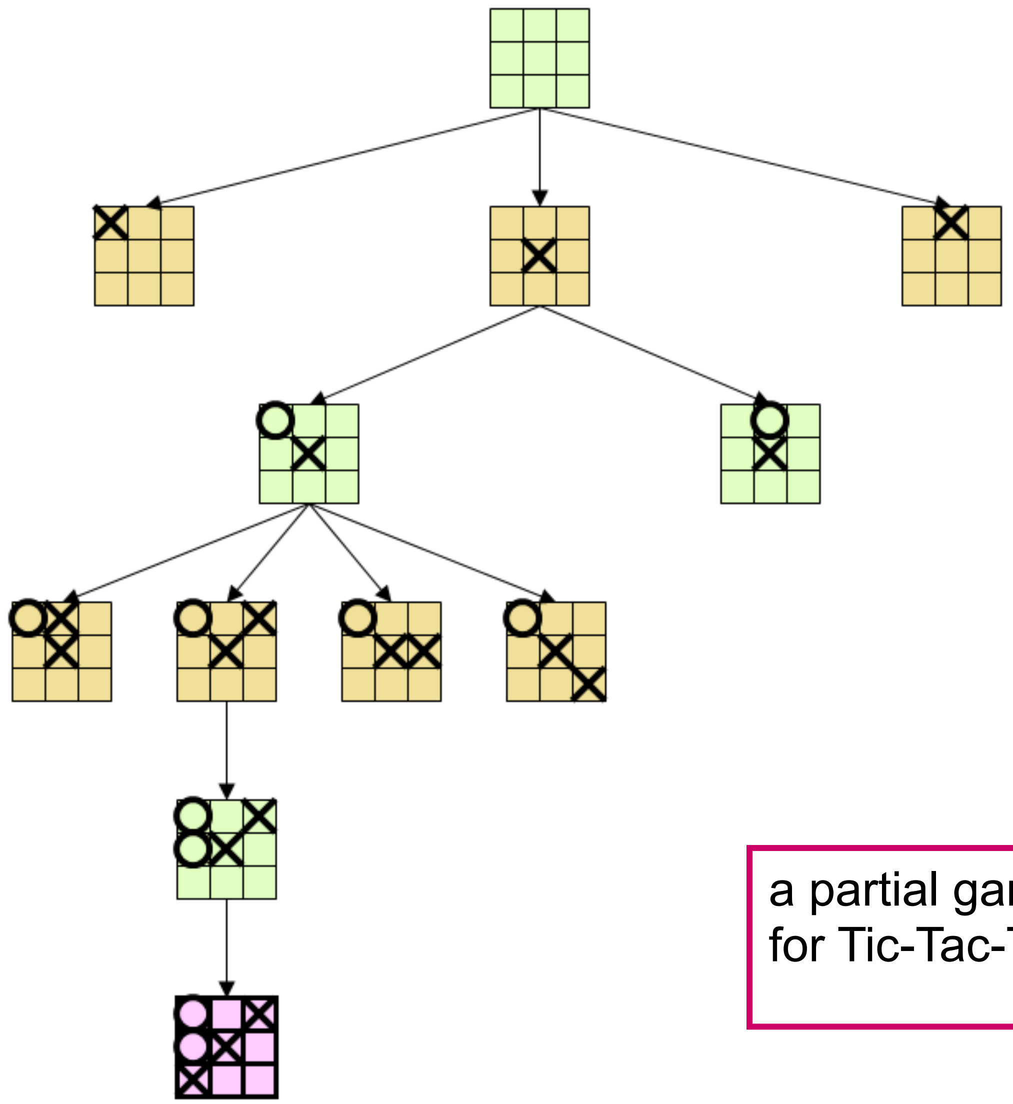
  - Is this what Deep Blue used?
  - What are some problems with this?
- More complex evaluation functions may involve <u>learning</u>
  - Adjusting weights based on outcomes
  - Perhaps non-linear functions
  - How to choose the *features*?

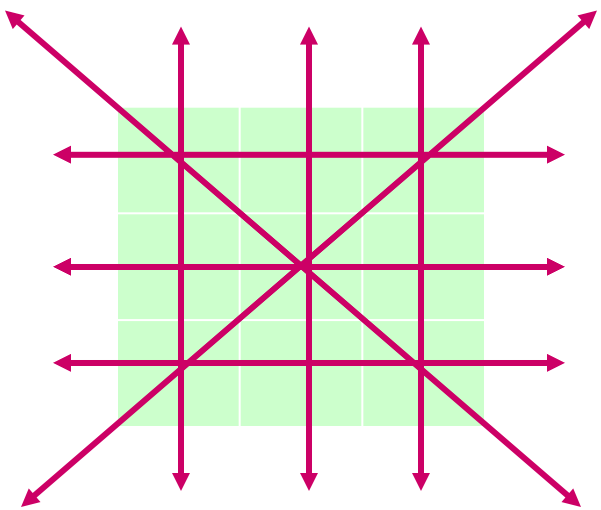
Tic-Tac-Toe revisited



a partial game tree
for Tic-Tac-Toe

Evaluation function for Tic-Tac-Toe

- A simple evaluation function for Tic-Tac-Toe
  - count the number of rows where **X** can win
  - subtract the number of rows where **O** can win
- Value of evaluation function at start of game is zero
  - on an empty game board there are 8 possible winning rows for both **X** and **O**

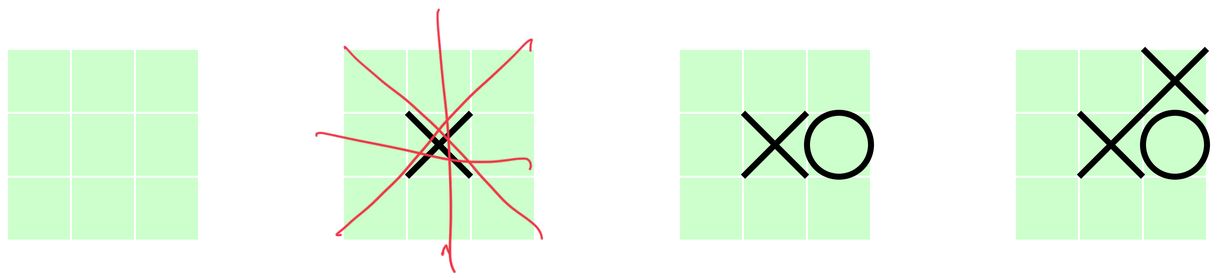


8-8 = 0

Evaluating Tic-Tac-Toe

```
evalX = (number of rows where X can win)  –
                (number of rows where O can win)
```

- After **X** moves in center, score for **X** is +4
- After **O** moves, score for **X** is +2
- After **X**'s next move, score for **X** is +4



8-8 = 0   8-4 = 4   6-4 = 2   6-2 = 4

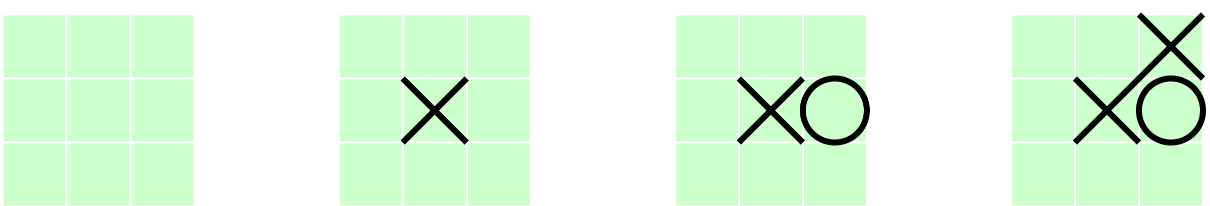
O

# of ways X can win

Evaluating Tic-Tac-Toe

```
evalO = (number of rows where O can win)  –
              (number of rows where X can win)
```

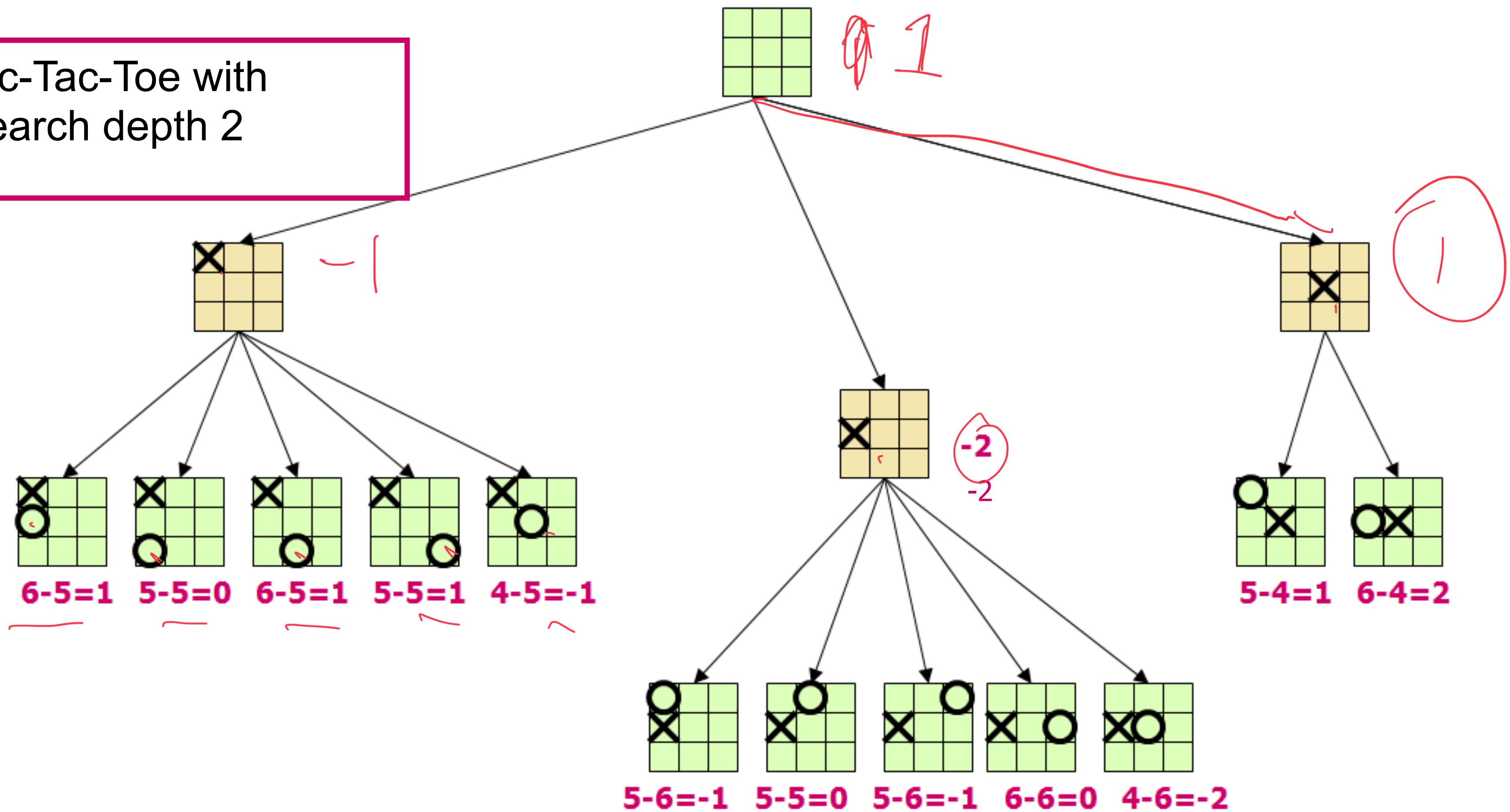
- After **X** moves in center, score for **O** is -4
- After **O** moves, score for **O** is +2
- After **X**'s next move, score for **O** is -4



8-8 = 0   4-8 = -4   4-6 = -2   2-6 = -4

Search depth cutoff

Tic-Tac-Toe with search depth 2



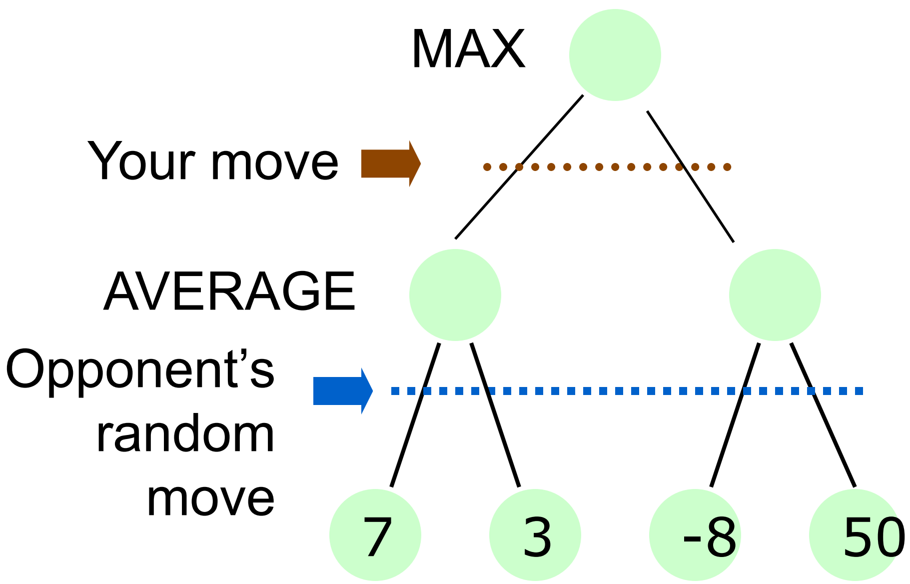
Evaluations shown for X

Expectimax: Playing against a benign opponent

- Sometimes your opponents are not clever.
  - They behave randomly.
  - You can take advantage of that by modeling your opponent.

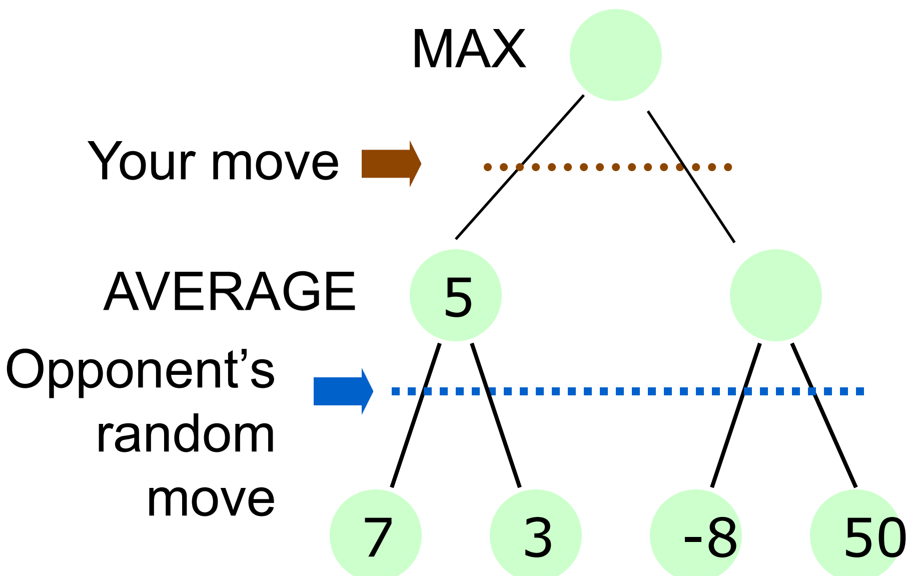
- Example of game of chance:
  - Slot machines
  - Tetris

Expectimax example



- Your opponent behave randomly with a given probability distribution,
- If you move left, your opponent will select actions with probability [0.5,0.5]
- If you move right, your opponent will select actions with [0.6,0.4]

Expectimax example



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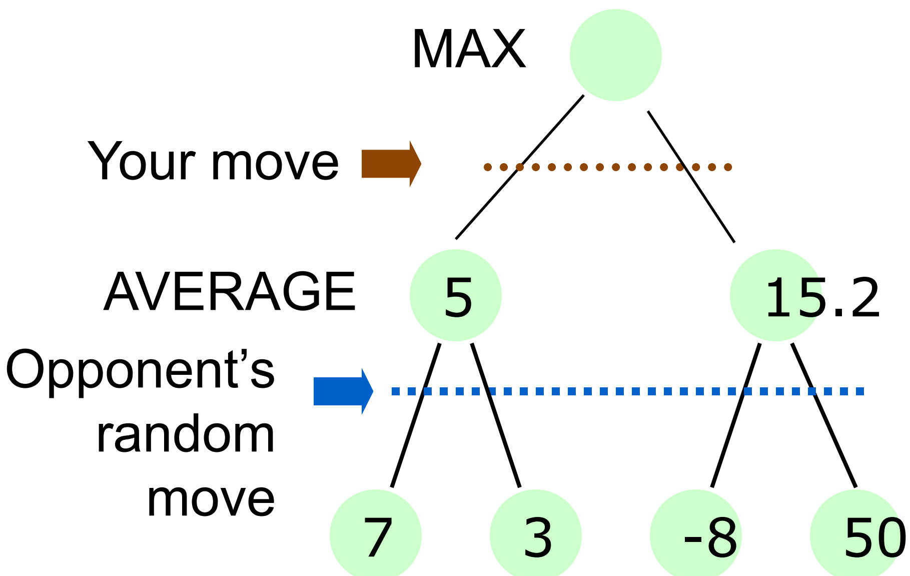

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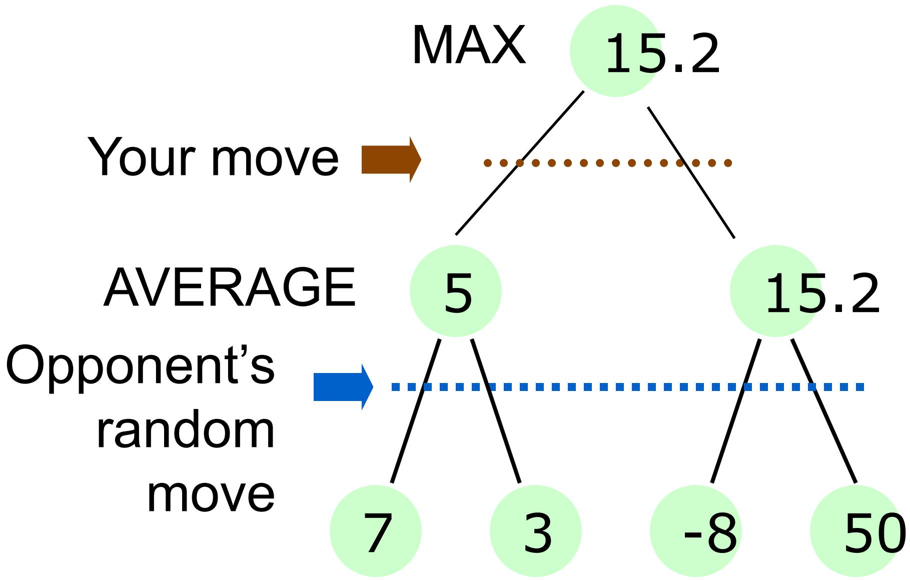

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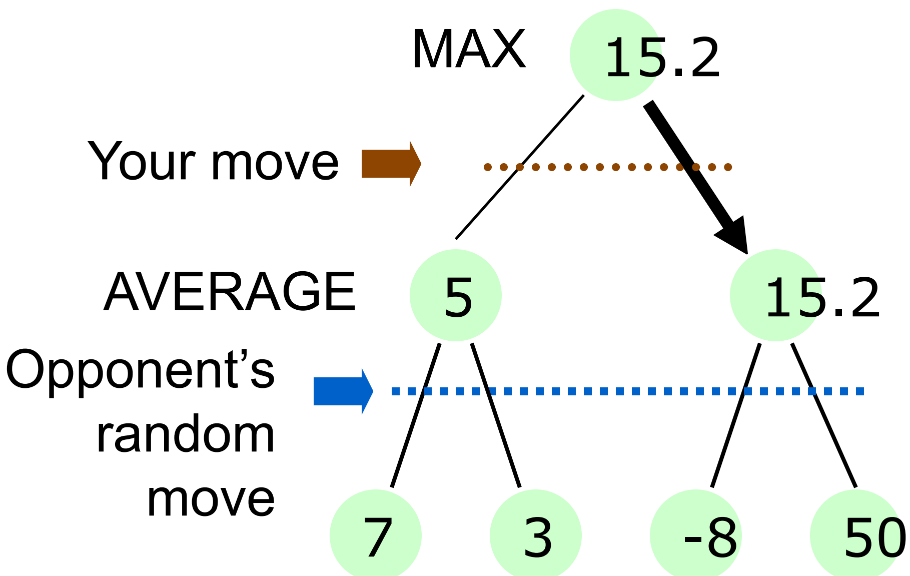

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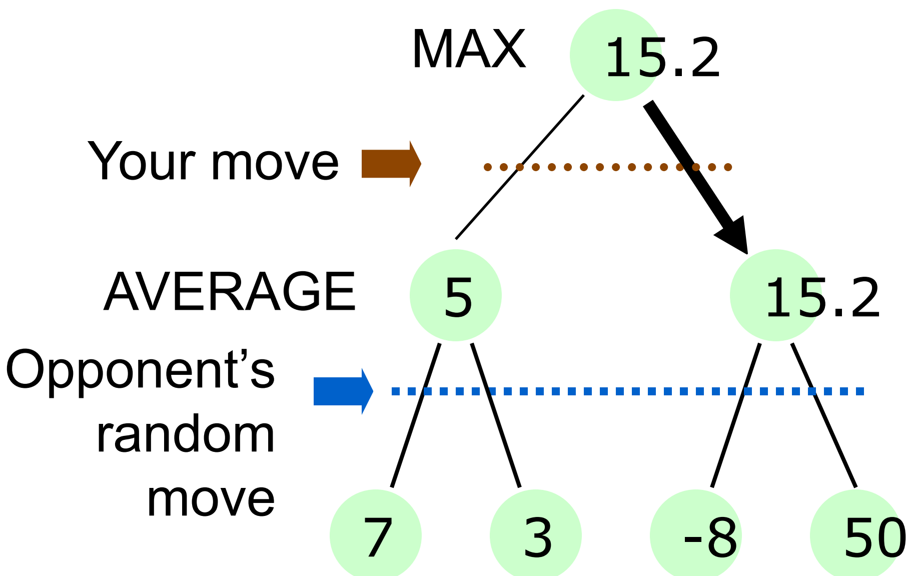

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Note: pruning becomes tricky in expectimax… think about why.

Summary of game playing

- Minimax search
- Game tree
- Alpha-beta pruning
- Early stop with an evaluation function
- Expectimax

More reading / resources about game playing

- Required reading: AIMA 5.1-5.3

- Stochastic game / Expectiminimax: AIMA 5.5
  - Backgammon. TD-Gammon
  - Blackjack, Poker

- Famous game AI: Read AIMA Ch. 5.7 (or in the "Historical notes" of the AIMA 4th Edition)
  - Deep blue
  - TD Gammon

- AlphaGo: https://www.nature.com/articles/nature16961