CS 136: MID-QUARTER EXAMINATION 2

Department of Computer Science
University of California, Santa Barbara
Closed-Book, 75 minutes

Fall 2002

INSTRUCTIONS

• Before you answer any questions, print your name and perm number.

• Read each question carefully. Make sure that you clearly understand each question before answering it.

• Put your answer to each question on its own page.

• You may wish to work out an answer on scratch paper before writing it on your answer page; answers that are difficult to read may lose points for that reason.

• There are 8 questions of equal value.
1. Let $w = s_1 s_2 \cdots s_n$, for $s_i \in \Sigma$. Define $w^R = s_n s_{n-1} \cdots s_1$. Let $L$ be a language. Define $L^R = \{ w^R \mid w \in L \}$.

If $L$ is regular, is $L^R$ regular? Prove your answer.
2. Let $A$ be an FA with 3 states over $\{a, b, c\}$.

Can you list the words that are sufficient to test for acceptance to see if $L(A) = \emptyset$? If yes, do so. If not, explain.

How many words need to be tested for acceptance to confirm that $|L(A)| < \infty$?
3. Let $L_1$ and $L_2$ be languages over $\{0, 1\}$. Prove or disprove the following statement:
   If $L_1 \subseteq L_2$ and $L_1$ is not regular, then $L_2$ is not regular.
4. Let $L_1$ and $L_2$ be languages over $\{0, 1\}$. Prove or disprove the following statement:

If $L_1$ and $L_2$ are not regular, then $L_1 \cup L_2$ is not regular.
5. Prove or disprove the following statement: If you subtract a finite set of words from a non-regular set, the result is a non-regular set.
6. Let $L_1$ and $L_2$ be context-free languages over the same set of terminals. Prove that $L_1 \cup L_2$ is a context-free language.
7. Let the set of terminals be \( \{a, b\} \). Give a context-free grammar for the set of words containing exactly 2 \( b \)'s or exactly 3 \( b \)'s.
8. Let $L = \{ww \mid w \in \Sigma^*\}$. Prove that $L$ is regular or prove that it is not regular.