Please read the corresponding chapter before attending this lecture.

These notes are not complete. They are supplemented with figures and material that arises during the lecture in response to questions.

Please report any errors in these notes to me (cappello@cs.ucsb.edu). I’ll fix them immediately.

*Based on An Introduction to Formal Languages and Automata, 3rd Ed., Peter Linz, Jones and Bartlett Publishers, Inc.*
2.2 Nondeterministic Finite Acceptors

Definition of a Nondeterministic Finite Acceptor

Defn. 2.4 A nondeterministic finite acceptor (NFA) is defined by a 5-tuple

\[ M = (Q, \Sigma, \delta, q_0, F), \]

where \( Q, \Sigma, q_0, F \) are defined as for DFAs, but

\[ \delta : Q \times (\Sigma \cup \{\lambda\}) \rightarrow 2^Q. \]

This definition differs from that for a DFA in 3 ways:

- The value of the transition function is a set of successor states.
- The set of successor states may be empty: There is no successor state.
- The transition function allows moves “on \( \lambda \)” that do not advance the read head.
Transition graphs for NFAs are suitably modified:

- If
  \[ \delta(q, a) = \{s_1, s_2, \ldots, s_k\}, \]
  where each \( s_i \in Q \), then there is an arc labelled \( a \) (including \( \lambda \)) from the node labelled \( q \) to each of the nodes labelled \( s_1, s_2, \ldots, s_k \).

Figure 1: An NFA

- Transitions that are unspecified implicitly indicate a transition to the empty set: *There is no successor state*, illustrated in Fig. 2.

Figure 2: An NFA that accepts \( \{01^n : n \geq 0\} \).

The transition function, \( \delta \), is extended analogously. We describe it in terms of transition graphs:
Defn. 2.5 For an NFA, the extended transition function is defined so that \( \delta^* (q, w) \) contains \( q' \) if there is a path in the TG from \( q \) to \( q' \) labelled \( w \), \( \forall q, q' \in Q, w \in \Sigma^* \).

For the NFA given in Fig. 3, what is:

- \( \delta^* (q_0, a) ? \)
- \( \delta^* (q_1, \lambda) ? \)
- \( \delta^* (q_1, a) ? \)
- \( \delta^* (q_2, a) ? \)
- \( \delta^* (q_0, b) ? \)

Figure 3: An NFA for \( \{a^n : n > 0\} \).

Informally, a word \( w \) is accepted by an NFA if there is a path labelled \( w \) from its initial state to some final state.
Defn. 2.6 The language $L(M)$ accepted by NFA $M = (Q, \Sigma, \delta, q_0, F)$ is
\[
\{ w \in \Sigma^* : \delta^*(q_0, w) \cap F \neq \emptyset \}.
\]

**Why Nondeterminism?**

Although many reasons may be put forth, 1 reason is that choices are represented by nondeterminism **succinctly**.

For example, given NFA $M_1$ and NFA $M_2$, construct an NFA $M_3$ such that $L(M_3) = L(M_1) \cup L(M_2)$. 