Please read the corresponding chapter before attending this lecture.

These notes are not intended to be complete. They are supplemented with figures, and material that arises during the lecture period in response to questions.

*Based on An Introduction to Formal Languages and Automata, 3rd Ed., Peter Linz, Jones and Bartlett Publishers, Inc.*
2.1 Deterministic Finite Acceptors

Deterministic Acceptors & Transition Graphs

Defn. 2.1 A deterministic finite acceptor (DFA) is defined by a 5-tuple

\[ M = (Q, \Sigma, \delta, q_0, F), \]

where

- \( Q \) is a finite set of states,
- \( \Sigma \) is a finite alphabet,
- \( \delta : Q \times \Sigma \mapsto Q \) is a total function called the state transition function,
- \( q_0 \in Q \) is the initial state,
- \( F \subseteq Q \) is a set of final states.
(Illustrate the operation of a DFA; see Java code for generic DFA.)

A **transition graph** (TG) is an alternate specification of a DFA:

- For each $q \in Q$, construct a node in the transition graph labelled $q$.
- For each $\delta(q, a) = q'$, construct an arc from the node labelled $q$ to the node labelled $q'$, labelled $a$.
- Distinguish the initial state with an incoming arc.
- Signify final states with a double circle.

Draw the transition graph that corresponds to the following DFA specification:

$$M = (\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_0\})$$

where $\delta$ is given by Table 1:

**Notation:**

$\lambda$ is the string of 0 symbols.
Table 1: Specification for a simple DFA.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>$q_0$</td>
<td>$q_1$</td>
</tr>
<tr>
<td>$q_1$</td>
<td>$q_1$</td>
<td>$q_2$</td>
</tr>
<tr>
<td>$q_2$</td>
<td>$q_2$</td>
<td>$q_2$</td>
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</tbody>
</table>

$\Sigma^* = \{\text{all finite strings over } \Sigma, \text{ including } \lambda\}.$

$\Sigma^+ = \Sigma^* - \{\lambda\}.$

We introduce the extended transition function,

$$\delta^* : Q \times \Sigma^* \mapsto Q :$$

$$\delta^*(q, \lambda) = q,$$  \hspace{1cm} (1)

$$\delta^*(q, wa) = \delta(\delta^*(q, w), a)$$  \hspace{1cm} (2)

for all $q \in Q, w \in \Sigma^*, a \in \Sigma.$
Heuristic: If an object is defined recursively, prove its properties by induction.

(Illustrate $\delta^*$ using the transition graph corresponding to Table 1.)

Languages & Dfa’s

Defn. 2.2 The language accepted by a DFA $M = (Q, \Sigma, \delta, q_0, F)$, denoted $L(M)$, is

$$\{w \in \Sigma^* : \delta(q_0, w) \in F\}.$$  

Observe that

$$\overline{L(M)} = \Sigma^* - L = \{w \in \Sigma^* : \delta(q_0, w) \notin F\}.$$  

(Illustrate notation for labelling 1 arc in a TG with multiple symbols in $\Sigma$.)

(Give a simple DFA; ask what language it accepts.)

(Illustrate a dead state aka trap state.)
Custom: When an arc from a TG node is missing (for 1 or more symbols) from a node, it is implicit that such transitions are to an implicit dead state.

Give a transition graph for \( \{01^n : n \geq 0\} \).

**Thm. 2.1** Let \( M = (Q, \Sigma, \delta, q_0, F) \) be a DFA, and let \( G_M \) be its associated TG. Then, \( \forall q, q' \in Q \) and \( w \in \Sigma^+, \delta^*(q, w) = q' \Leftrightarrow \exists \) a \([q, q']\) path in \( G_M \) labelled \( w \).

**Proof**

We prove this by induction on \( |w| \).

**Basis:** If \( w \in \Sigma \), then, by the construction of \( G_M \), \( \delta^*(q, w) = \delta(q, w) = q' \Leftrightarrow \) the TG has an arc labelled \( w \) from the node labelled \( q \) to the node labelled \( q' \).

**Induction hypothesis:** Assume, when \( |w| = n \) that \( \forall q, q' \in Q \) and \( w \in \Sigma^+, \delta^*(q, w) = q' \Leftrightarrow \exists \) a \([q, q']\) path in \( G_M \) labelled \( w \).
**Induction step:** We now show, when $|w| = n + 1$ that $\forall q, q' \in Q$ and $w \in \Sigma^+$, $\delta^*(q, w) = q' \iff \exists$ a $[q, q']$ path in $G_M$ labelled $w$.

Let $w = va$, where $|v| = n$ and $a \in \Sigma$.

By the induction hypothesis, $\forall q, q'' \in Q$, $\delta^*(q, v) = q'' \iff \exists$ a $[q, q'']$ path in $G_M$ labelled $v$.

Let $p = \delta^*(q, v)$.

Then, by the induction hypothesis, the TG has an arc labelled $v$ from the node labelled $q$ to the node labelled $p$.

Case $\delta(p, a) = q'$:

$\delta^*(q, w) = \delta(\delta^*(q, v), a) = \delta(p, a) = q'$ and the TG has an arc labelled $v$ from the node labelled $q$ to the node labelled $p$, and, by construction of the $G_M$, there is an arc labelled $a$ from the node labelled $p$ to the node labelled $q'$. 
Case $\delta(p, a) \neq q'$:
\[ \delta^*(q, w) = \delta(\delta^*(q, v), a) = \delta(p, a) \neq q' \] and the TG has an arc labelled $v$ from the node labelled $q$ to the node labelled $p$, and, by construction of the $G^*_M$, there is no arc labelled $a$ from the node labelled $p$ to the node labelled $q'$.

This completes the proof by induction on $|w|$.

Problem: Give a TG for:

1. $\{w \in \{0, 1, 2\}^* : w.\#(0) \equiv 1 \text{ mod } 2\}$.
2. $\{w \in \{0, 1, 2\}^* : w.\#(0) \equiv 1 \text{ mod } 2 \text{ and } w.\#(1) \equiv 0 \text{ mod } 2\}$.
3. $\{w \in \{0, 1, 2\}^* : w.\#(0) \equiv 1 \text{ mod } 2 \text{ and } w.\#(1) \equiv 0 \text{ mod } 2 \text{ and } w.\#(2) \equiv 1 \text{ mod } 3\}$. 