Dimensionality Reduction for Similarity Searching in Dynamic Databases

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Abstract

Databases are increasingly being used to store multi-media objects such as maps, images, audio and video. Storage and retrieval of these objects is accomplished using multi-dimensional index structures such as R^*-trees and SS-trees. As dimensionality increases, query performance in these index structures degrades. This phenomenon, generally referred to as the dimensionality curse, can be circumvented by reducing the dimensionality of the data. Such a reduction is however accompanied by a loss of precision of query results. Current techniques such as QBIC use SVD transform-based dimensionality reduction to ensure high query precision. The drawback of this approach is that SVD is expensive to compute, and therefore not readily applicable to dynamic databases. In this paper, we propose novel techniques for performing SVD-based dimensionality reduction in dynamic databases. When the data distribution changes considerably so as to degrade query precision, we recompute the SVD transform and incorporate it in the existing index structure. For recomputing the SVD-transform, we propose a novel technique that uses aggregate *data from the existing index rather than the entire data. This technique reduces the SVD-computation time without compromising query precision. We then explore efficient ways to incorporate the recomputed SVD-transform in the existing index structure without degrading subsequent query response times. These techniques reduce the computation time by a factor of 20 in experiments on color and texture image vectors. The error due to approximate computation of SVD is less than 10%.*

1 Introduction

Rapid advances in information technology have opened up new frontiers for research and development in the past decade. One such area is the storage and management of multi-media objects such as maps, images, text, audio and video. A natural approach for retrieving such multi-media objects is by their content. In this content-based retrieval, data objects *similar in content* to a query object are retrieved. Over the years, such content-based retrieval of images, video, audio text and time sequences has received much attention in the database community. To support such similarity queries, similarity metrics between different objects are maintained in the database. For most applications such as images, text or time sequences, the similarity metrics can be embedded in a high-dimensional vector space. Proximity in the high-dimensional vector space reflects the similarity of the content of the data objects in these applications. For instance, the color content of images may be described by the percentage of pixels in the image that have a particular color. The different possible colors form different dimensions, and the computed color fractions form the values along these dimensions. Given this transformation, a request for images similar to a query image can be answered by searching in the proximity of the query image in the high-dimensional space. This approach for content-based retrieval reduces the problem to one of searching in high-dimensional space. In this paper, we explore how to support this type of search efficiently for dynamic databases, where objects may be inserted and deleted frequently.

Searching in multiple dimensions has been extensively researched in the database and computational geometry literature. Several data structures such as R-trees [1, 2, 13], hB-trees [18], Bang files [7, 8, 9], TV-trees [17], SS-trees [24], and SR-trees [16] are designed for supporting fast searching in large multi-dimensional databases. These structures are quite efficient for small dimensions (of the order of 1-10). However, as the data dimensionality increases, the query performance of these structures degrades rapidly. For instance, White and Jain [24] report that as the dimensionality increases from 5 to 10, the performance of a nearest-neighbor query in multi-dimensional structures such as the SS-tree [24] and the R*-tree [1], degrades by a factor of 12. This phenomenon, appropriately termed the *dimensionality curse* by Agrawal and Faloutsos [6], is a common characteristic of all multi-dimensional index structures. In spite of the progress in the design and analysis of multi-dimensional structures such as the TV-trees [17], the X-trees [2], the SS-trees [24], and the SR-trees [16], the dimensionality curse persists.

The only known solution for achieving scalable query performance is by reducing the dimensionality of the data. Several research initiatives [5, 14, 17, 20, 21] including the QBIC project adopt this approach for supporting image retrieval based on their content. This approach for content-based retrieval first condenses most of the information in a dataset to a few dimensions by applying Singular Value Decomposition (SVD). The data in the few condensed dimensions are then indexed to support fast retrieval based on their color, texture or shape content. This approach assumes that the SVD is computed *a priori* on the entire data. Therefore, it works only for static databases and is not applicable to dynamic databases. In this paper, we focus on dynamic databases, where the SVD needs to be recomputed due to changes in the underlying data. We propose a novel technique to efficiently recompute SVD for such dynamic environments.

Much research has been done on updating SVD as new vectors are added to, and old ones deleted from a database [3, 4, 12, 19]. A simple technique [11] for computing SVD after a new item is added to a database of n d-dimensional vectors has $O(n * d^2)$ time complexity. However, incremental techniques [3, 4] reduce this complexity to $O(n * r)$, where r is the rank of the matrix of the n d-dimensional vectors (r is usually much smaller than d). Although these techniques reduce the multiplicative factor from $O(d^2)$ to $O(r)$, the time complexity is still linear in the number of vectors n. In this paper, we propose a novel approach to reduce this linear dependence: instead of using the entire set of n vectors, we use an *aggregate* data set of m vectors, $m << n$, to recompute the SVD. This technique reduces the recomputation time, irrespective of the SVD update algorithm used. The quality of the recomputed SVD depends on how well the aggregate data set reflects the data distribution. We ensure this by obtaining the aggregate data set from the higher layers of an index constructed for that data. This technique achieves high precision of query results with low SVD-recomputation times. In our experiments, we observe that this approach reduces the computation time by a factor of up to 20, while admitting an approximation error of less than 10%. This approach is quite elegant for large databases, where not all data can reside in memory.

We then examine how to incorporate the recomputed SVD in the existing reduced-dimensional index structure. Incorporation of the SVD transform in the leaf entries of the index is relatively straightforward. Incorporation in the index entries, however, is more complex because of the high overheads and the potential for increasing query response times. We examined three different approaches for incorporation of SVD-transform in index entries. The first approach reconstructs the entire index. Although this approach ensures good query response time, it results in high incorporation times for disk-resident indices. The second approach reuses the existing index and only adjusts the extents of the index entries. This approach achieves low incorporation times but poor query performance since the index may not reflect the data distribution after transformation. The third approach selectively reconstructs subtrees at a chosen level and reuses the existing index for higher levels. The level of reconstruction can be chosen based on the availability of memory resources. This approach combines the merits of both the previous approaches – it ensures good performance of subsequent queries and achieves low incorporation overheads.

The rest of the paper is organized as follows. Section 2 surveys dimensionality-reduction techniques and shows that SVD-based dimensionality reduction is superior to other approaches. Section 3 describes how to efficiently recompute SVD-transform using an existing index structure. Section 4 describes how to efficiently incorporate the recomputed transform function in an index structure. These results are summarized in the final section.

2 Background

In this section, we examine the implications of dimensionality reduction specifically the loss of information and the subsequent loss in query accuracy. We then describe several techniques to ensure high query accuracy during dimensionality reduction. These techniques are only applicable to static databases where data items are known *a priori*. We then present our technique for ensuring high query accuracy in dynamic environments.

2.1 Implications of Dimensionality Reduction: Loss in Query Accuracy

Reducing data dimensionality may result in a loss of information. To illustrate this, let us consider the example of Figure 1. The two nearest neighbors of point A are B and C. However, if we retain only the x-coordinates of the points, then the two closest points for A are B and D. Likewise, if we retain only the y-coordinates, then point F becomes one of the two closest neighbors of point A. In either case, there is a loss of proximity information as we move from 2 dimensions to 1 dimension. This loss in distance information due to dimensionality reduction is usually measured in terms of precision and recall of a random query point [22]. This is detailed next.

Let A_d denote a set of d-dimensional points. Let A_k denote the set of points in k-dimensional space, where $k \leq d$. Let $R(q, A_d)$ denote the set of data points satisfying a query q in the d-dimensional space and let $R(q, A_k)$ denote the query result in the k-dimensional space. Then, two measures of query accuracy can be defined as follows.

1. Precision(q, d, k) = $\frac{|R(q,A_k) \cap R(q,A_d)|}{|R(q,A_k)|}$.
2. Recall(q, d, k) $=$ $\frac{|R(q,A_k) \cap R(q,A_d)|}{|R(q,A_d)|}$.

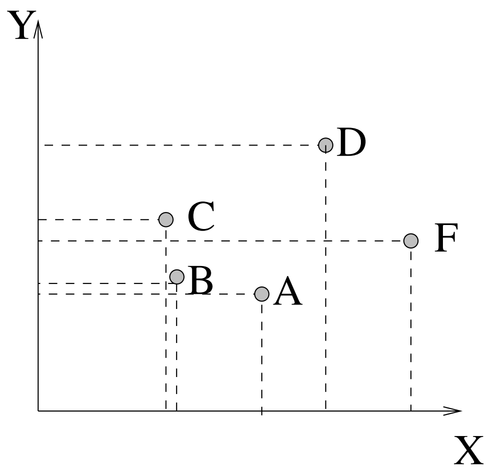


Figure 1. Example of 2-dimensional data: Data points are represented by circles.

Note that for a query retrieving the m-nearest neighbors (m-nearest neighbor query), $|R_d(q)| = |R_k(q)| = m$. Therefore, the precision and recall of a nearest neighbor query are identical. In this paper, we only concentrate on nearest neighbor queries since these are the most common types of queries on high-dimensional data. For the example data of Figure 1, the precision of a 2-nearest-neighbor query located at point A is 0.5 using either the x-dimension or the y-dimension. In general, the most varying dimensions are retained to minimize the loss of distance information measured using precision and recall. However, the information in these dimensions may not be high enough to achieve good precision and recall (as seen in Figure 1). In what follows, we see how transforming the data first and then reducing the number of dimensions might help.

2.2 Improving Query Accuracy using Data Transformations

The data may be first transformed so that most of the information gets concentrated in a few dimensions. These dimensions are then identified and the coordinates of the data items in these dimensions are subsequently used for indexing to support fast searching. We refer to such dimensions as the *chosen* dimensions. For example, if the axes are rotated from (X, Y) to (X_1, Y_1) as shown in Figure 2, and X_1 is chosen as the dimension for indexing, then the 2-nearest-neighbor relationships of all points including A are preserved.

Example transformations include Singular Value Decomposition (SVD), Discrete Fourier Transform (DFT), Discrete Cosine Transform (DCT), Wavelet Transform and Harr Transform. Of these, the SVD technique examines the entire data and rotates the axes to maximize variance along the first few dimensions. In contrast, DFT, DCT, Wavelets and Harr transforms process each individual data point separately. These transforms are used in the context of time series database [6] and highly correlated image databases [25]. However, for most data sets, SVD explicitly tries to maximize the variance in the first few dimensions, and therefore, achieves higher precision and recall than other transforms. Figure 3 illustrates this for a database of 144K 256-dimensional color histograms. This figure compares the query precision for a 21-nearest-neighbor query using SVD and DFT transforms. For the SVD-transformed data, the first few dimensions are retained. For the DFT-transformed data, the most varying dimensions are retained and

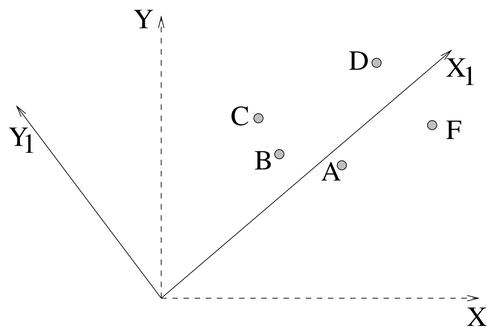


Figure 2. Axes change from (X, Y) to (X_1, Y_1).

is referred to as *VDFT* (Variance DFT) in the figure[1]. The figure plots the average query precision for 2000 random queries when the number of retained dimensions is varied between 32 and 256. We observe that SVD achieves around 50% higher precision than DFT. This explains why SVD is the default choice for dimensionality reduction of high-dimensional data in most research and commercial systems such as QBIC [21] and LSI [14]. Next, we describe SVD-transformation scheme in more detail and examine how it is incorporated in static databases.

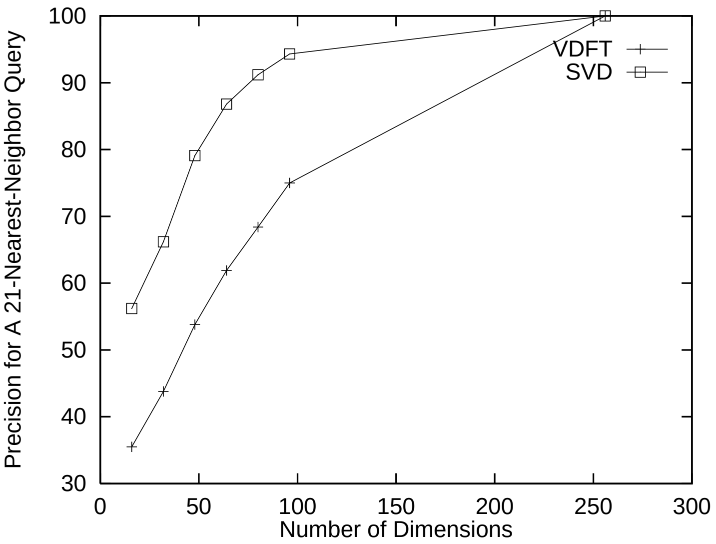


Figure 3. Precision for 21-nearest-neighbor queries as the number of chosen dimensions is varied for SVD- and DFT-transformed data. Database: 144K color histograms. Original dimensionality: 256.

[1] Alternative choices for dimensions such as the most *dominating* dimensions (DDFT) [25] achieve similar results to those using most varying dimensions.

2.2.1 Singular Value Decomposition

A set A of n d-dimensional vectors can be transformed using SVD as follows.

- Compute the d-by-d *SVD-transformation Matrix V*. This matrix is determined by decomposing matrix A as follows:

$$A = U \Sigma V^T$$

  Here, A is the $n \times d$ data matrix composed of the n d-dimensional vectors, U is an $n \times d$ matrix, Σ is a $d \times d$ singular value matrix and V is a $d \times d$ orthonormal *basis* matrix. We refer to the basis matrix as the SVD-transform matrix.

- The transformed data is given as $U\Sigma$. It is computed by multiplying each vector P in the original data matrix A with the basis matrix V.

In contrast to DFT and other transforms, computation of the SVD transform matrix V takes $O(n * d^2)$ time, and therefore, is quite expensive. Note that the time is linear in the number of points, but the constants involved are quite high.

2.2.2 Current Techniques for Incorporating SVD-transform in Index Structures

Current techniques [20, 21, 14] for supporting content-based retrieval on text documents, time sequences, or color, texture, shape features of images employ the following approach to combine fast query response times with high accuracy of query results. First, the original d-dimensional data is transformed using SVD. For this, first the SVD-transform matrix is determined using the entire data. Then, it is applied on each individual vector separately to obtain the transformed d-dimensional vectors. In the second step, the d-dimensional data is reduced to k-dimensional data by retaining only the coordinates in the first k dimensions. The first k dimensions are chosen since most information is concentrated in these dimensions. The reduced-dimensionality data are then indexed using multi-dimensional index structures such as R*-trees, SS-trees [24], and SR-trees [16]. Queries are also transformed using the SVD transform matrix and and answered using the reduced-dimensional index structure.

To illustrate this with an example, consider the example data of Figure 1. This figure shows five 2-dimensional points. To reduce the dimensionality to 1 and index them using a 1-dimensional structure to achieve fast query responses, the following actions are taken. The 2-dimensional data is first transformed using SVD. As a result, the axes are reoriented from (X, Y) to (X_1, Y_1) as shown in Figure 2. Most of the proximity information is concentrated in the X_1-dimension. The coordinates of all the points in this dimension are used in constructing a 1-dimensional index. Figure 4 shows the index structure thus formed. The extent of the index entries I_1 and I_2 is also shown along the X_1 axis in the figure. Note that the entries in the leaf level point to the actual 2-dimensional data items. We refer to these entries as *data* entries. All other entries point to index nodes, and therefore, are referred to as *index* entries. Queries are first transformed using SVD and then processed using the index entries of the reduced-dimensional index. The data entries may then be used to retrieve the actual data items (for instance, the browse graphics in case of image data).

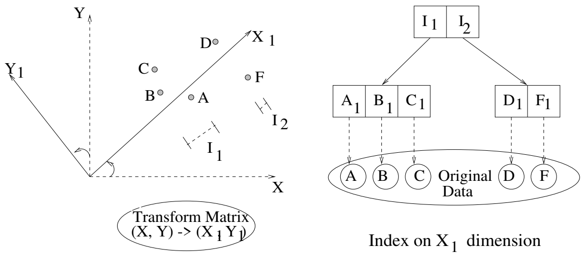


Figure 4. A 1-dimensional index structure for the 2-dimensional data items A, B, C, D and F. The index structure is constructed using the coordinates of the data items along axis X_1.

This technique works well for static databases. However, in dynamic databases, where inserts and deletes occur frequently in the database, the axes may have to be reoriented to cope with these frequent updates to the database. Otherwise, the query precision may drop considerably. Figure 5 illustrates this by measuring the query precision at various times during the creation of an index. At first a small database of 2K 256-dimensional color histograms are considered. The SVD transform matrix is computed for these data objects. Subsequent inserts to the database are also transformed using the same SVD transform matrix. The figure plots the query precision achieved as the database grows. This precision is then compared with the precision achieved by recomputing the SVD after every update (shown by the boxes on the "SVD-recomputed" curve). Each of these numbers for query precision are computed as the average of two thousand 21-nearest-neighbor queries. The query points are randomly picked from the set of points in the database. The x-axis plots the size of the database. The y-axis plots the query precision achieved. We notice that as the database grows, the query precision drops if the SVD is not recomputed. This is because the SVD that was computed for 2K points becomes outdated as new data points are inserted. In practice, data items may also be deleted rendering the SVD transform outdated after a small number of updates.

2.3 Improving Query Accuracy in Dynamic Databases

In this subsection, we describe our approach for dimensionality reduction in dynamic databases, where insertions and deletions occur frequently. To cope with such insertions and deletions, we suggest that the SVD transform be recomputed whenever it becomes ineffective due to changes to the underlying data distribution. Specifically, the query accuracy may be monitored and an SVD-recomputation triggered whenever the accuracy drops below acceptable levels. For most image databases supporting content-based retrieval, a small drop of 5-10% could be tolerated. Alternatively, a significant drop in the variance in the chosen dimensions (as a percentage of the total variance) may be used to trigger an SVD-recomputation. Such a recomputation strategy would improve the overall query accuracy of the system and maintain it at acceptable levels. For instance, Figure 6 shows the rise in query precision with the SVD being recomputed first when the database grows by a factor of 8 and then when the database grows by a factor of 64.

In this paper, we assume that the SVD-recomputation is triggered by some system-defined criteria such as drop in query precision, or drop in the variance in the chosen dimensions. In what follows, we explore efficient techniques for accomplishing this recomputation and incorporation in an index structure. As in QBIC, we assume that a reduced-dimensional index structure is created by first computing the SVD-transform for a small data set. As new data items

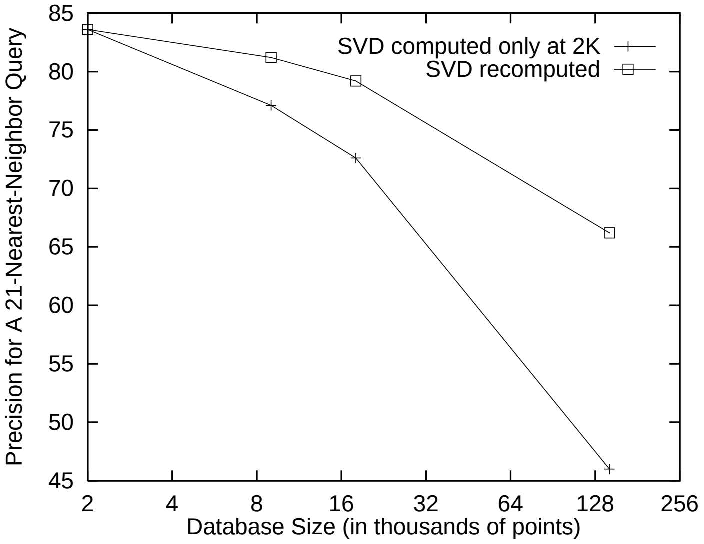


Figure 5. Drop in query precision for a 21-nearest-neighbor query with the growth of a 256-dimensional color histogram database: Total size is 144K. Reduced-dimensionality is 32. Boxes indicate where recomputation is done for "SVD-recomputed" curve.

are inserted and old ones deleted, the SVD-transform (basis matrix) has to be first recomputed, and then incorporated in the index structure.

3 Recomputing SVD Transform Matrix

In this section, we describe how to recompute the SVD-transform in a dynamic index. We assume the index stores all data items at the leaf level. We consider two principal approaches for computing the SVD transform matrix. The first approach uses the entire data to determine this matrix. The second approach uses an aggregate data set which is much smaller than the original database to reduce computation overheads.

3.1 SVD Recomputation using Entire Data

Consider the example data of Figure 4. Assume four new data items G, H, K and L are added to the database of Figure 2. The data distribution after the four insertions is shown in Figure 7. Each of these data items is first reoriented using the transform matrix from the (X, Y) domain to (X_1, Y_1) domain. The coordinates along the X_1 dimension are chosen for insertion in the index. A pointer is maintained from the leaf entry where it is inserted in the index to the complete data item in the original domain in the database. This is illustrated in Figure 8. The extents of the index entries are also indicated in the figure.

Let us assume that the SVD-transform matrix is to be recomputed at this time. This could be accomplished as follows.

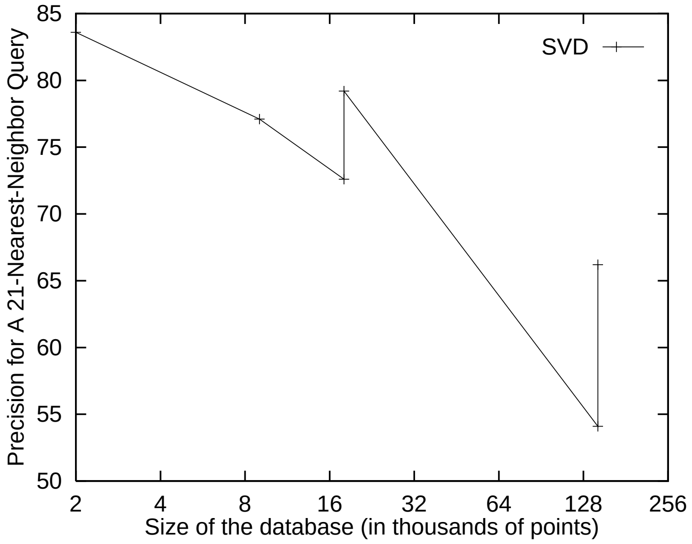


Figure 6. Improvement in query precision of Figure 5 due to SVD-recomputation: Recomputation occurs when the database size is 18K and 144K.

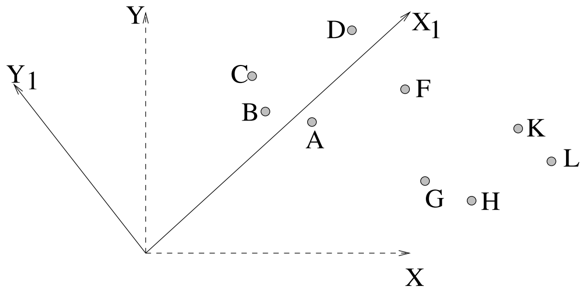


Figure 7. Data after adding 4 new items G, H, K and L.

Step 1 (Data Access): Access all the data objects A, B, C, D, F, G, H, K and L through the leaves of the index.

Step 2 (SVD Computation): Determine the SVD-transformation matrix for the data items.

Since the transform has to be computed over the entire dataset, we refer to this technique as the *All-Data-SVD*, or simply as the *SVD* technique, where there is no ambiguity. Since the time overhead for computing SVD in step 2 is directly proportional to the number of points used in the computation, the overhead is quite large for large datasets and dominates the total overhead.

3.2 SVD Recomputation using Aggregate Data

In order to reduce the time for SVD computation, we use an aggregate data set whose size is much smaller than the size of the database. The aggregation is performed based on the structure of the existing index. This allows us to

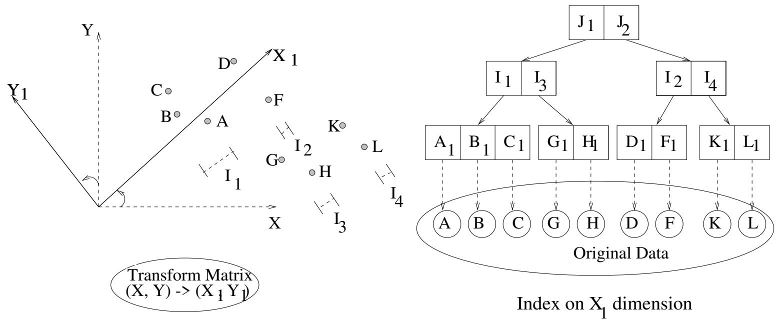


Figure 8. Index structure after adding 4 new data items G, H, K and L.

capture a good approximate distribution of the data. Consequently, SVD-transform computed using the aggregate data set is quite close to that computed using the entire data. We refer to this as *Approximate-SVD* computation. The whole recomputation process involves the following two steps.

Step 1 (Aggregate Dataset Extraction): Obtain an *aggregate set* of data points that reflects the data distribution.

Step 2 (SVD-Computation): Compute the SVD-transform matrix using the aggregate data.

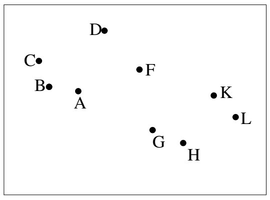
To obtain the aggregate data, we choose a level of the index structure, and then for each subtree rooted at that level, we access all its data items and compute their centroid. The centroids of all the subtrees at the chosen level constitute the *aggregate* data set. Note that as we move up the tree, the number of subtrees and therefore the size of the aggregate dataset decreases, leading to a large reduction in SVD-computation time. However, the resulting aggregate data set may capture less of the nuances of the underlying data distribution, thereby leading to large approximation errors. In practical database systems, most tree structures have a height of 3, or at most 4 due to large node capacities. By picking the aggregate data set from the level just above the leaf level in these large-node-capacity index structures, we obtain a considerable reduction in SVD computation time without introducing high approximation errors.

To illustrate this idea, let us consider the example data of Figure 8. In Figure 9, we plot the data used for computing SVD in the All-Data-SVD technique and the Approximate-SVD technique. This data is shown as filled holes in the figure. Note that although the Approximate-SVD technique uses only 4 points for computing SVD instead of 9 as in All-Data-SVD, it still reflects the approximate distribution of the data. Both data sets obtain similar re-orientations for the axes due to near-identical data distribution characteristics.

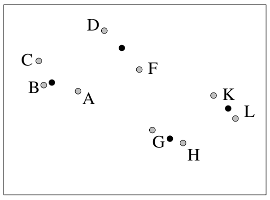
In what follows, we describe some experiments to evaluate the approximate SVD-recomputation technique. Here, we measure how effective the SVD-transform matrix computed by Approximate-SVD is in comparison to one obtained from the entire data (All-Data-SVD). We measure this effectiveness by comparing the query precision obtained using both the schemes.

3.3 Experiments

We experimented with the above approach for SVD-recomputation using an SS-tree index structure. The SS-tree is implemented in C on an SGI Indy Machine with 192 MB of main memory running IRIX 5.3 Operating System. The



(a) All-Data-SVD



(b) Approximate-SVD

Figure 9. Data samples used in computing SVD: Black dots indicate points used in the computation.

page size of the SS-tree is fixed at 8K. The capacity of the SS-trees, therefore, depended on the number of dimensions being indexed. For instance, for 12-dimensions, the node capacity was 120 whereas for 32-dimensions the capacity was 55. The re-insertion parameter of the SS-tree is set to 30% of the node capacity and the minimum capacity to 40% of the node capacity. These are the same as in other implementations of SS-trees [24] and R*-trees [1]. To simulate real-database scenarios, only 25% nodes of an index are cached in memory in all our experiments. The caching strategy is simple and ensured that all the non-leaf nodes are in memory along with first few leaf nodes that are created.

We evaluate the effectiveness of the approximate computation of the SVD on two real image databases and one synthetic dataset. The first real image dataset is the Corel Image Collection, which consists of 26K 48-dimensional feature vectors representing image textures. The second one is an assortment of 18K 256-dimensional color histograms obtained from the WWW.

For each database, we first compute the SVD-transform for a fraction ($1/8^{th}$) of the database, reduce the dimensionality and then construct an index for the reduced-dimensional data. We then insert the remaining points of the database into the index. Then, we recompute the SVD-transforms using both aggregate data (as in Approximate-SVD) and the entire data (as in All-Data-SVD). The computation times for determining the transform are measured. The quality of the transforms is compared by evaluating the average query precision for 2000 nearest-neighbor queries. The query points are randomly chosen from among the data points in the database. In what follows, we describe the results for the two databases.

3.3.1 Texture Dataset

For the texture database, we assume that the SVD-transform is computed when the database has approximately 3K data items (corresponding to $1/8^{th}$ of the total database size of 26K). We then insert 23K more data points into this index and then determine the SVD-transform matrix using Approximate-SVD and All-Data-SVD techniques. We measure the average query precision achieved when the resulting SVD-transform matrices are applied to the original 48-dimensional data.

First, we examine the computation times of the two techniques. We note that the cost of obtaining the aggregate data in Approximate-SVD is the same as that of All-Data-SVD. Since this cost is very small (15 seconds) in comparison to the SVD-computation times, we only compare the SVD-computation times for the two techniques. Table 1 shows these computation times. Note that as we increase the number of dimensions, the capacity of the index nodes decreases.

Consequently, the ratio of the aggregate data set to that of the database also increases since the aggregate data set is obtained from the parents of leaf nodes in an index. This leads to an increase in the SVD computation time for the Approximate-SVD. However, the computation time is still smaller by around two orders of magnitude than the time for computing using the entire data set.

Dimensions	Approximate-SVD		SVD	
	Points	Time (s)	Points	Time (s)
8	205	1.6	26K	209
12	272	2.1	26K	209
16	333	2.65	26K	209
20	403	3.1	26K	209

Table 1. SVD-computation times using Approximate-SVD and All-Data-SVD (SVD) techniques. Database: 26K 48-dimensional texture vectors.

Next, we evaluate the query precision achieved using the approximate technique. Figures 10 and 11 show the average precisions for nearest-neighbor queries using the transform matrices computed by the All-Data-SVD and Approximate-SVD techniques. Figure 10 plots the query precision for different number of nearest neighbors assuming a reduced dimensionality of 12. Figure 11 plots the query precision for a varying number of reduced dimensions and 21-nearest-neighbor queries. We observe that Approximate-SVD achieves nearly the same query precision as SVD. This is significant given the efficiency of computing Approximate-SVD as opposed to SVD. In contrast, other techniques such as DFT (also plotted) do not achieve as high a precision.

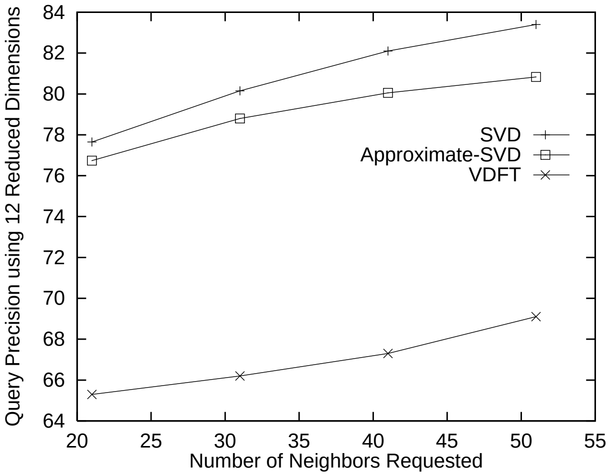


Figure 10. Precision of nearest-neighbor queries on 26K 48-dimensional texture vectors: Varying number of neighbors requested for 12 dimensions.

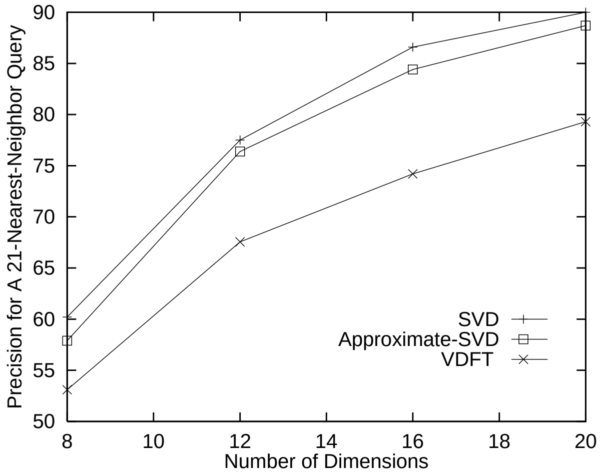


Figure 11. Precision of nearest-neighbor queries on 26K 48-dimensional texture vectors: Varying number of chosen dimensions for 21-nearest-neighbor queries.

3.3.2 Color Dataset

For the color database, we assume that the SVD-transform is computed when the database has 2K entries (corresponding again to $1/8^{th}$ of the database size), and add another 16K entries using this transform. The SVD-transform is recomputed using both Approximate-SVD and All-Data-SVD techniques. As before, we compared the relative merits of the two techniques by examining the computation time and query precision achieved.

Table 2 shows the computation times for Approximate-SVD and All-Data-SVD. As the dimensionality of the index increases from 16 to 64, the node capacity decreases. This leads to an increase in the size of the aggregate dataset (because of the increase in the number of subtrees at the lowest non-leaf level), thereby leading to an increasing in SVD-computation times. We observe that the computation time for Approximate-SVD is less by a factor of 25 than that of All-Data-SVD.

Dimensions	Approximate-SVD		SVD	
	Points	Time (s)	Points	Time (s)
16	236	55	18K	5013
32	416	102	18K	5013
48	632	154	18K	5013
64	821	201	18K	5013

Table 2. SVD-computation times using Approximate-SVD and All-Data-SVD (SVD) techniques. Database: 18K 256-dimensional color histograms.

Figures 12 and 13 plot the query precision obtained for this dataset. As before, we consider a varying number of nearest neighbors as well as a varying number of reduced dimensions. The results are similar to those for the Texture dataset. Once again we observe that the query precision using Approximate-SVD is within 5% of the query precision using SVD, and that the query precision using DFT (VDFT) is much lower.

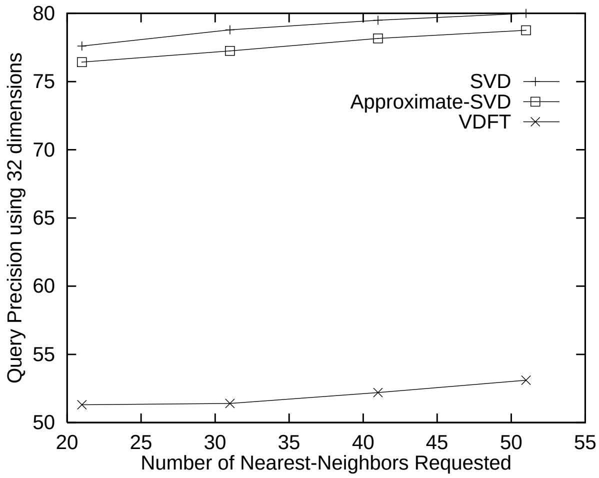


Figure 12. Precision for nearest-neighbor queries on 18K 256-dimensional color histograms: Varying number of neighbors requested for 32 dimensions.

Experiments with Aggregation from different levels of index

In the above experiments, we evaluated the quality of approximate-SVD assuming the aggregate data is obtained from the level just above the leaves. Next, we examine the impact of obtaining aggregate data from higher levels of the index on the quality of approximate-SVD. We examine two levels of the index one just above the index (level 1), and another just two levels above the leaves (level 2). Using the aggregate data at each of these levels, we compute the approximate-SVD transform and compare their query precision. We conduct these experiments on a 144K 256-dimensional color histogram database, whose SVD is first computed when the database has only 18K histograms.

Figure 14 illustrates the query precision for 32 reduced dimensions. We observe that as we move up the tree for obtaining the aggregate data, the query precision drops. This is expected as the nuances in the underlying data distribution are lost as we move to higher levels of an index structure. Similar results are also obtained in figure 15 where the number of reduced dimensions is varied from 16 to 64.

3.4 Experiments with a synthetic dataset

Next we describe our experiments on a synthetic dataset. The dataset is generated as follows. The dimensionality of the dataset is chosen to be 100. Initially, 5K data points are generated so that the variance is nearly the same in all the dimensions: for each dimension, the range of the domain is fixed as [0,1] and data values are randomly generated

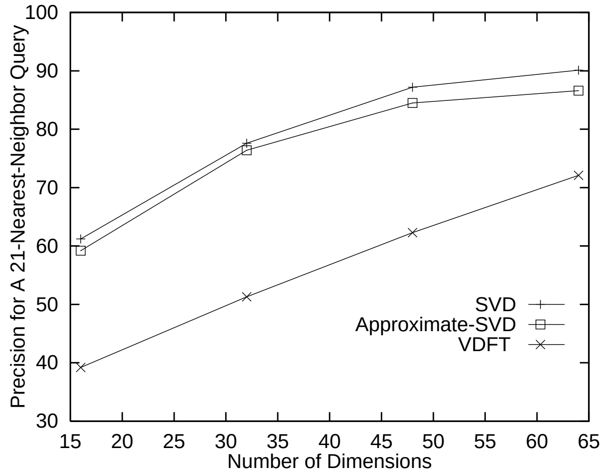


Figure 13. Precision for nearest-neighbor queries on 18K 256-dimensional color histograms: Varying number of chosen dimensions for 21-nearest-neighbor queries.

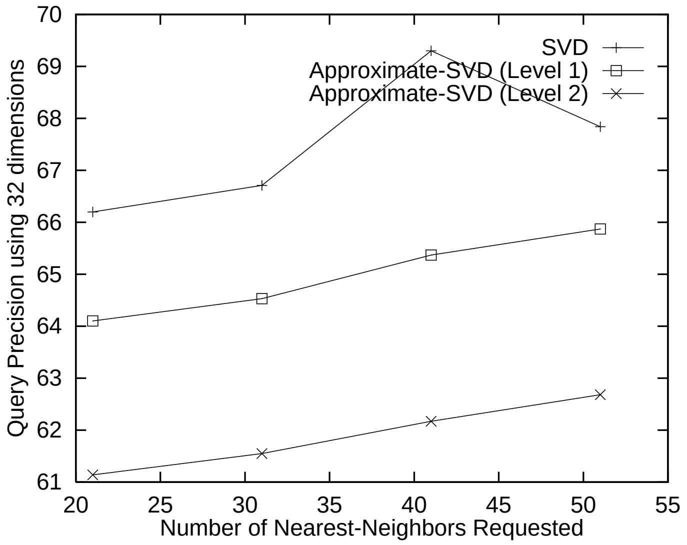


Figure 14. Precision for nearest-neighbor queries on 144K 256-dimensional color histograms: Varying number of neighbors requested for 32 dimensions. Approximate-SVD is computed from level 1 and level 2 of index.

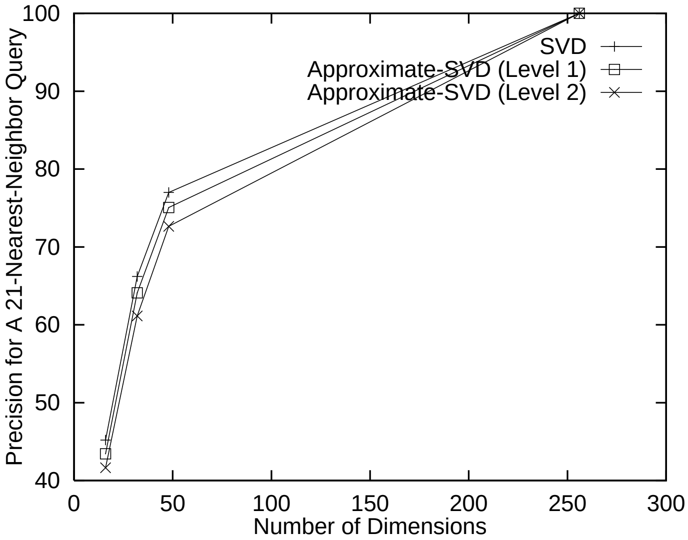


Figure 15. Precision for nearest-neighbor queries on 144K 256-dimensional color histograms: Varying number of chosen dimensions for 21-nearest-neighbor queries. Approximate-SVD is computed from level 1 and level 2 of index.

in this range. We then increase the database size to 40K (as in the previous experiments) by adding 35K more points. For these 35K points, the variance is decreased by half for every 20 dimensions: the points in the first 20 dimensions range over the domain [0,1], those in the next 20 dimensions range over [0, 0.5], and so on.

Given this synthetic dataset, the SVD transform in computed for the first 5K data points. A reduced-dimensional index is constructed using the first 20 dimensions after this data transformation. Subsequently, the remaining 35K points are then inserted in the index. The SVD-transform is recomputed using the index. As in the previous cases, approximate computation of SVD using the index is upto two orders of magnitude faster than computation using entire data. Figures `16 and 17 describe the quality of approximate-SVD transform in comparison to SVD and DFT. These results indicate that the query precision achieved by approximate-SVD is comparable to that of SVD. Note that DFT achieves poor query precision for the same synthetic dataset.

In these experiments, we have observed that computing approximate-SVD using aggregate data from an existing reduced-dimensional index achieves good query precision while requiring low computation overheads. The approximate-SVD may, however, perform poorly if the clusters of the index do not reflect the underlying distribution. This may happen due to a poor choice of the dimensions for the reduced-dimensional index based on initial database characteristics. In such a case, the index may be reconstructed using the current "most-varying" set of dimensions, and then the clusters from this index may be used for computing approximate-SVD transform.

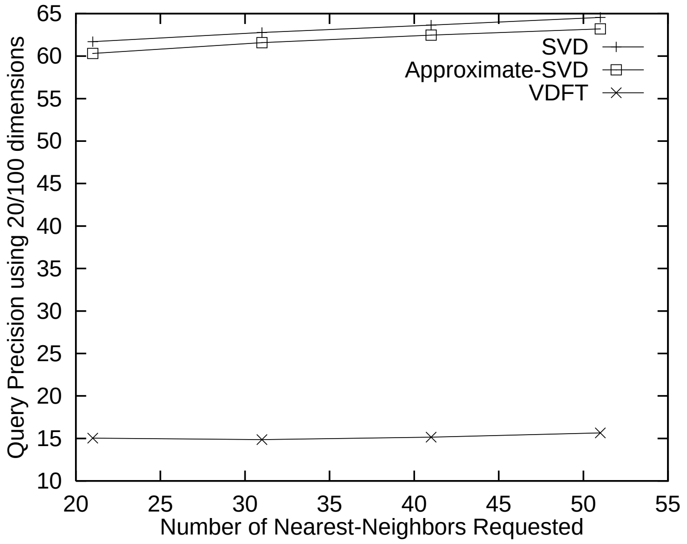


Figure 16. Precision for nearest-neighbor queries on 40K 100-dimensional vectors: Varying number of neighbors requested for 20 dimensions.

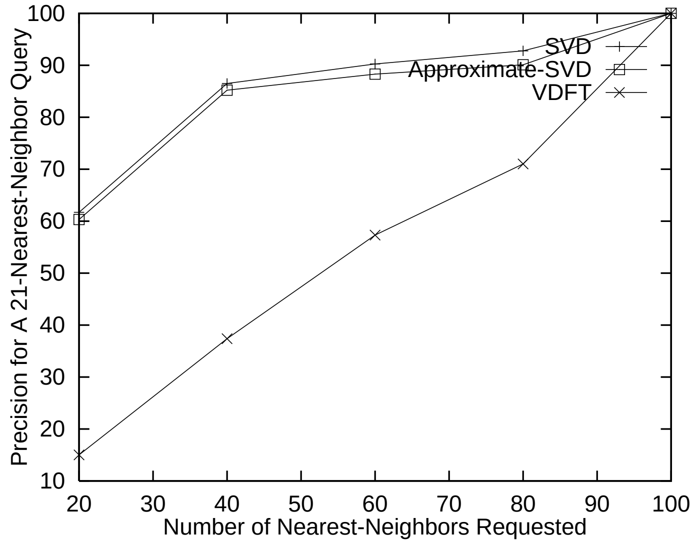


Figure 17. Precision for nearest-neighbor queries on 40K 100-dimensional vectors: Varying number of chosen dimensions for 21-nearest-neighbor queries.

4 Incorporating Recomputed SVD-transform in Reduced-Dimensional Index

In the previous section, we examined how to determine the SVD-transform matrix during recomputation. In this section, we explore how to incorporate the transform matrix in the existing reduced-dimensional index structure.

Incorporating the recomputed SVD transform at the leaf level entries is relatively straightforward: just apply the new transform individually to each data item and store the reduced dimensions in the leaf level entries of the index. Incorporation is much more complex for the index entries, since this has a direct impact on subsequent query performance. We discuss below two obvious approaches to this incorporation. The first, called *Tree-Reconstruct* or simply as *Reconstruct*, reconstructs the entire index afresh. The second approach, called *Structure-Reuse* or simply as *Reuse*, reuses the existing index structure and adjusts the extents of the index entries without any structural changes. Finally, we detail our approach that combines the benefits of the two previous approaches. This scheme called *Reuse-Reconstruct* performed much better in our experiments.

The *Reconstruct* strategy reconstructs the entire index from the data entries. If the data can reside entirely in memory, this reconstruction can be accomplished efficiently using the techniques of in-memory index structures such as k-d trees and its variants [15, 10, 23]. For disk-resident datasets (like what we used in our experiments), the reconstruction has to be done a few data items at a time, leading to high insertion costs. On the positive side, this technique achieves a good query response time since the index structure reflects the data distribution accurately.

The *Structure-Reuse* strategy reuses the existing index structure and recomputes the extent of the index entries without making any structural changes. The extent of an index entry is recomputed recursively using the extents of the child entries. These updates are carried out in a bottom-up manner starting with the leaf level. Since no structural changes are made, this incorporation strategy is quite efficient. However, the extents of the index entries are likely to increase due to reorientation of the SVD axes. As a result, query performance degrades. In what follows, we describe a third scheme that combines the fast query response times of a tree-reconstruction strategy and the low incorporation overheads of a structure-reuse strategy. Note that all three schemes have the same query precision, since they use the same SVD matrix for transforming the data items.

4.1 The Reconstruct-Reuse Approach

This approach for incorporation selectively reconstructs subtrees in the index and reuses the higher levels of the index. Figure 18 depicts this strategy for an arbitrary index. Based on the available memory resources, a specific level of the index is chosen for reconstruction. Above this chosen level, the index structure is reused as in the Structure-Reuse strategy, and below this level the index is recomputed as in the *Reconstruct* strategy. Choosing a level close to the root leads to more memory requirements since a larger subtree of the index has to be manipulated in the memory.

The reconstruction of a subtree is performed using the VAMSplit technique [23]. The set of data items in the subtree is partitioned recursively until each sub-partition fits in a single node (disk page). The partitioning is accomplished by determining the most varying dimension, and splitting the data along an appropriate place in this dimension. Given n data items to be partitioned into two sub-partitions, the number of data items in the first sub-partition, m, is computed based on a node capacity of b as follows.

$$m = \begin{cases} \lfloor \frac{n}{2} \rfloor \; if \; n \leq 2b \\ b \, \lfloor \frac{n}{2b} + 0.5 \rfloor \; otherwise. \end{cases}$$

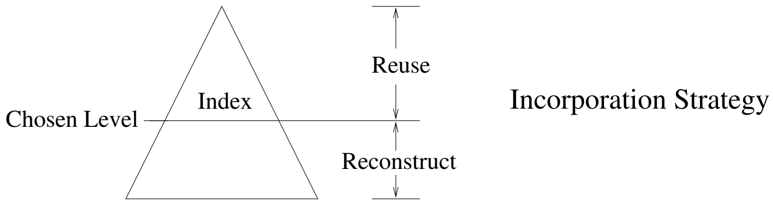


Figure 18. Reuse-Reconstruct Strategy for Incorporating SVD-transform in an Existing Index

White et al. [23] showed that this choice of m minimizes the number of nodes in the created index. They also showed that the resulting index supports fast queries.

The query performance of the *Reuse-Reconstruct* strategy described above is enhanced by selectively deleting some data entries during subtree reconstruction and reinserting them in the entire tree. This eliminates the impact of data entries that are *misfits* in a subtree after the transform matrix is applied. An entry is considered to be a misfit in a subtree if its deletion results in a considerable decrease in the volume. We found that this selective re-insertion improved the query performance.

4.2 Experimental comparison of the three approaches

We experimented with the above incorporation strategies on an image database of 26K 48-dimensional texture vectors. These strategies are used to incorporate a recomputed SVD-transform matrix in an existing SS-tree index for the texture data. We evaluated the incorporation strategies by measuring incorporation times and query response times after incorporation. The query response times are computed as the average over two thousand 21-nearest-neighbor queries. Each query is generated using a randomly picked data point from the database.

For the *Reuse-Reconstruct* scheme, we used selective reinserts. We assumed that only those data entries that resulted in a 50% radius decrease for a subtree are considered for deletion. The number of such reinserted entries is further limited to 5% of the entries in a subtree. For the *Reconstruct* scheme, we assumed the tree is reconstructed using an SS-tree [24]. In all the experiments, we assumed that only 25% of the index nodes are cached in memory.

Scheme	Incorporation time (s)	Query time (ms, disk I/O)
Reconstruct	1363	(27, 154)
Reuse	22	(52, 275)
Reuse-Reconstruct	42	(30, 160)

Table 3. Incorporation and 21-nearest-neighbor query response times for different schemes. Database: 26K 48-dimensional texture vectors. Reduced dimensionality: 12.

Table 3 compares the three incorporation schemes for the texture database. The best query response time is achieved by reconstructing the entire index as in the *Reconstruct* scheme. However, this scheme has a high incorporation cost.

On the other hand, the best incorporation time is obtained by reusing the existing structure as in the *Reuse* scheme. However, this scheme lead to a considerable increase in the extents of index entries. The average radius of the index entries actually tripled in this case. Consequently, the query response times nearly doubled. These two schemes form the two extremes. In comparison, the *Reuse-Reconstruct* scheme achieves a query response time that is within 10% of the best query response time registered (by the *Reconstruct* scheme). The incorporation time of the *Reuse-Reconstruct* scheme is double that of the *Reuse* scheme, though within acceptable limits and much lower than that of the *Reconstruct* scheme. The performance advantages of the *Reuse-Reconstruct* scheme are expected to be substantial even if other strategies are used for tree-reconstruction.

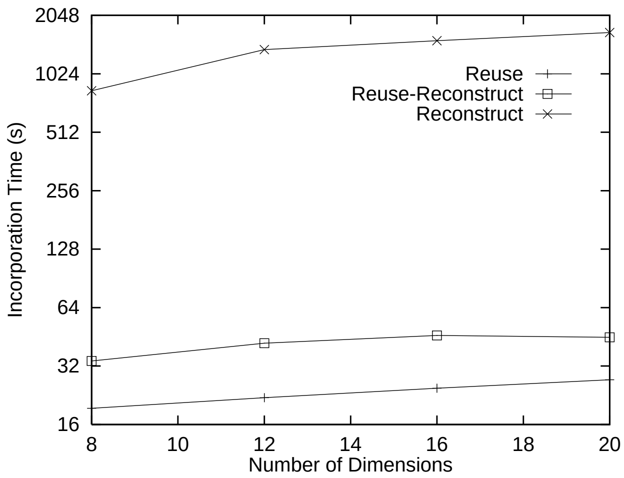


Figure 19. Incorporation times for differet incorporation schemes as number of chosen dimension is varied. Database: 26K 48-dimensional texture vectors.

Figures 19 and 20 elaborate these results for incorporation and query response times when the number of reduced dimensions is varied. The results are similar to the case of 12 dimensions in Table 3. Similar results were also obtained for color histograms. These results indicate that our proposed scheme, which combines low incorporation times of the *Reuse* scheme and the low query response time of the *Reconstruct* scheme, is a viable strategy for incorporating recomputed SVD transforms in dynamic databases.

5 Conclusions

Performance of similarity searching in high dimensional index structures degrades as the dimensionality of the data increases. Dimensionality reduction is the only known solution to support fast searching. Such dimensionality reduction is accompanied by a loss of information and a consequent loss in query accuracy. This loss is minimized if the data is first transformed using an SVD transform, which concentrates most of the information in a few dimensions. The reduced-dimensionality data obtained from an SVD-transformation could then be used to support fast searching with high query accuracy. Current systems using this approach perform a static transformation of the data. In this paper, we illustrate why such an approach may lead to degradation in query accuracy as data items are inserted and

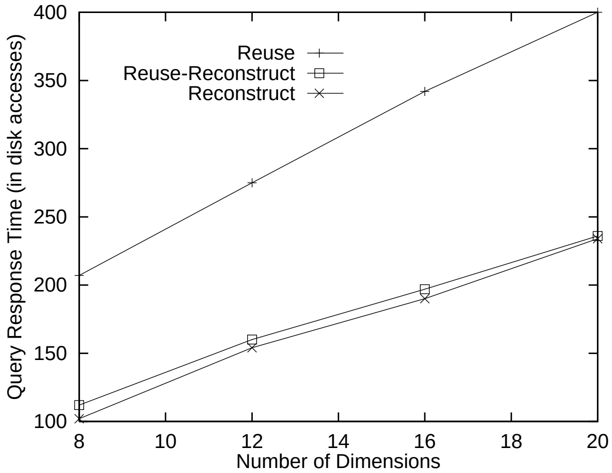


Figure 20. Query response times for different incorporation schemes as number of chosen dimensions is varied. Database: 26K 48-dimensional texture vectors.

Transform Comp. + Incorporation	Query Precision	Query time (disk I/O)	Accumulated SVD-time (s)	Accumulated Incorporation time (s)
Approximate-SVD + Reuse-Reconstruct	61.6	1390	916	184
SVD + Reconstruct	66.2	1270	40318	21370

Table 4. SVD-transform computation and incorporation times for 2 recomputations during insertion of 144K 256-dimensional color histograms. Query response time and query precision after the insertion process are also indicated.

deleted. We, therefore, propose novel techniques to cope up with the updates to the database.

First, we propose an efficient techniques for approximately computing SVD. In contrast to existing techniques, our technique uses a small aggregate dataset instead of the entire data to compute the SVD transform. Since the size of the aggregate dataset is quite small in comparison to that of the database, the overheads of computation are reduced by up to two orders of magnitude. The aggregate data is obtained from the index clusters and closely reflects the underlying data distribution. This results in approximation error of less than 5%.

We then explored how to efficiently incorporate the recomputed SVD-transform in the existing reduced-dimensional index structure. Straightforward approaches to incorporation of the transform in an index could degrade index quality and increase subsequent query response times. Therefore, we examined different ways to efficiently incorporate the transform matrix in the index without degrading query performance. We proposed a novel strategy that selectively reconstructs subtrees at a chosen level and reuses the index at higher levels. The level of reconstruction can be chosen to suit available memory resources.

We also ran experiments to measure the impact of multiple cycles of SVD recomputation and subsequent incorporation in an existing index. The experiments were conducted for 144K 256-dimensional color histograms. An initial All-Data-SVD was computed and an SS-tree index was constructed for 3K data items. Two recompute-incorporate instances were chosen at database sizes of 18K and 144K. The results comparing our techniques with existing ones are shown in Table 4. The total times for the SVD recomputations and subsequent incorporations, in addition to query precision and query response time are indicated in the table.

Note that the query precision using Approximate-SVD is within 9% of the precision using All-Data-SVD. The response time is also close to that of a completely reconstructed index. However, the reduction in SVD-computation time and incorporation times are around two orders of magnitude for our proposed techniques. We conclude that such huge reduction in computation overheads will contribute to high index performance both in single-user and concurrent systems. These experiments considered dynamic environments in which the size of the database grows by large amounts through insert operations. Similar shifts in SVD vectors can also be caused by a smaller number of updates (that include deletes). In a realistic setting, we expect that recomputation of SVD would be triggered either after a predetermined number of update operations, or by a loss in query precision. In future, we will investigate the robustness of our approach in the context of other types of queries such as epsilon queries, and other datasets.

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