Lecture 13
Deep Latent Models
Variational Inference

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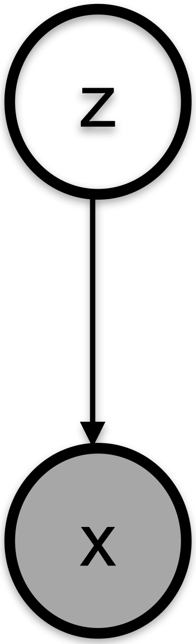
Final Project Presentation

- Poster Presentation, Dec 5, 10am-12:30pm.
- Clearly present
  - broad motivation / larger context
  - what is the problem you are trying to solve
  - why is it important
  - what is your novel contribution
  - experimental/theoretical validation
  - what are observations/discoveries
  - Takeaway message/insights.
- do not use too much text, instead put figures, tables, illustrations, examples.
- Good news: CS department will sponsor printing/post stand cost!

Deep Latent Model

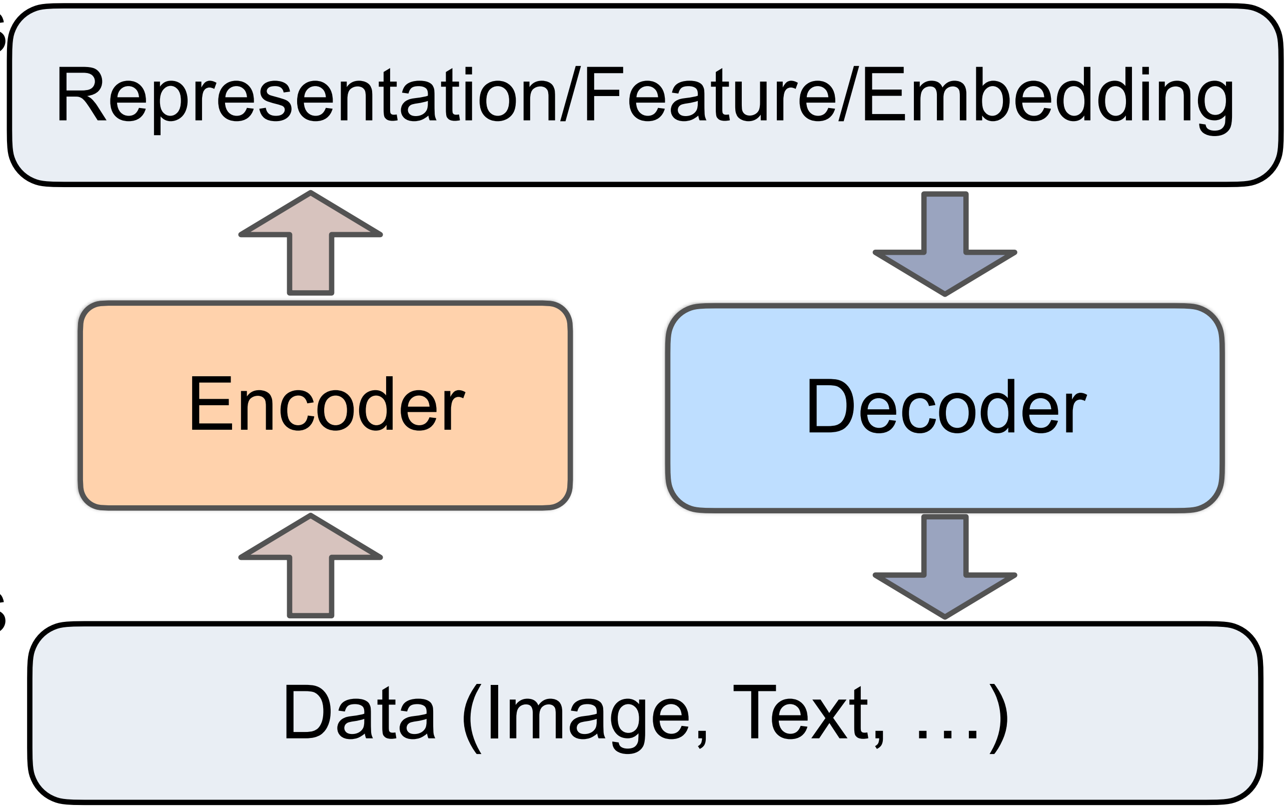
- z follows a prior distribution, e.g. Gaussian(0, I)
- p(x|z) is defined by a deep neural network $f(z;\theta)$
- To learn θ, use $E_{(Z|X)}[\log p(X,Z;\theta)]$



Variational Auto-Encoder (VAE)

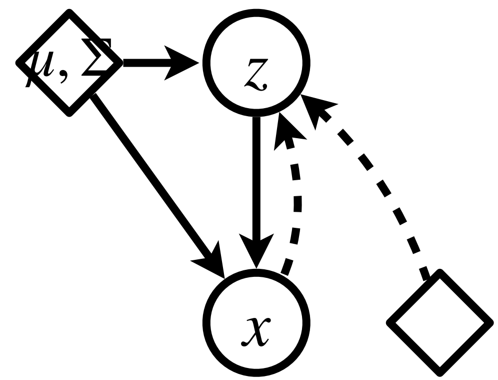
VAE

- Hidden representations follow a prior distribution
- Encoder will produce a distribution of representations (posterior distribution)



Graphical Model for VAE

- Assuming data X is generated from a latent variable Z
- Generation process
  - draw $Z \sim N(\mu, \Sigma)$
  - draw $X \mid Z \sim p(f(Z))$, defined by a neural network f



- The goal is to maximize the data log-likelihood

$$\log p(X;\theta) = \log \int p(X \mid Z)p(Z)dZ$$

- Hard to optimize over θ, if f(Z) is very complex such as a CNN, RNN, or Transformer.

Lower bound for VAE

Objective: maximize the data loglikelihood

$$\max \ell(\theta) = \sum_n \log p(x_n; \theta)$$

$$= \sum_n \log \int p(x_n \mid z_n; \theta) p(z_n; \theta) dz_n$$

Lower-bound

$$\max \ell(\theta) = \sum_n \log p(x_n; \theta)$$

$$= \sum_n \log \int p(x_n \mid z_n; \theta) p(z_n; \theta) dz_n$$

- But $\log p(x; \theta)$ is intractable.

$q(z \mid x; \phi)$ is the posterior distribution from encoder!

- For any distribution $q(z \mid x, \phi)$:

$$\log p(x; \theta) \geq \mathrm{E}_{q(z|x;\phi)} \left[\log \frac{p(x, z; \theta)}{q(z \mid x; \phi)} \right] = \mathrm{ELBO}$$

- Derivation via Jensen's inequality.
- Maximizing the ELBO instead of maximizing $\log p(x; \theta)$

Understanding ELBO

$$\log p(X;\theta) \geq \mathrm{E}_q[\log \frac{p(X,Z;\theta)}{q(Z \mid X;\phi)}]$$

$$\max_\theta \max_\phi \mathrm{ELBO} = \sum_n \mathrm{E}_q \left[\log \frac{p(x_n \mid z_n;\theta) p_0(z_n)}{q(z_n \mid x_n;\phi)}\right]$$

$$=$$

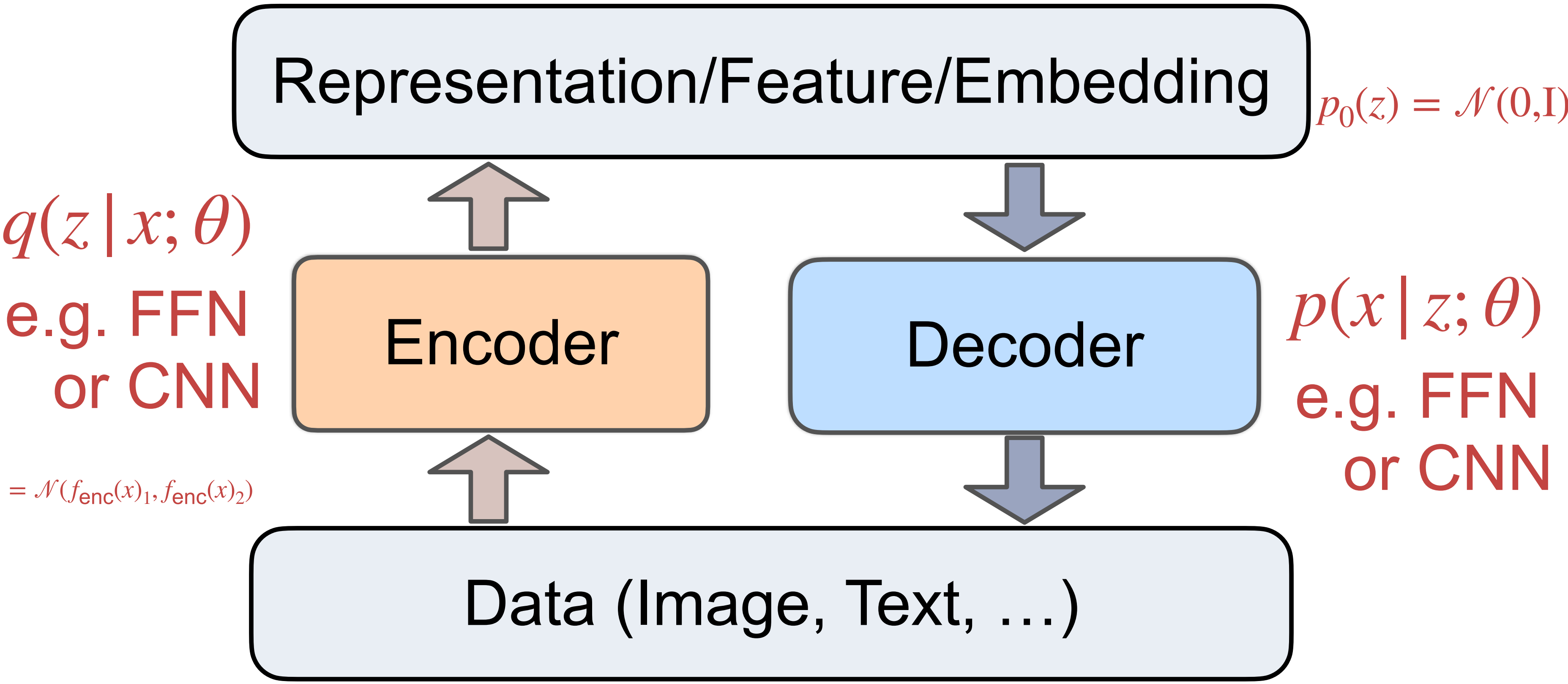
$$=$$

$$= \mathrm{E}_q \left[\log p(x_n \mid z_n;\theta)\right] - \mathrm{KL}\left(q(z_n \mid x_n;\phi) \| p_0(z_n)\right)$$

Reconstruction loss Regularization

VAE

Let $q(z \mid x; \phi)$ and $p(x \mid z; \theta)$ share the same parameter θ



Training VAE

gradient descent(ascent for max)

$$\max_{\theta} \max_{\phi} \text{ELBO} = \sum_{n} \mathrm{E}_{q(z_n|x_n;\theta)} \left[\log \frac{p(x_n \mid z_n; \theta) p_0(z_n)}{q(z_n \mid x_n; \theta)} \right]$$

$$= \sum_{n} \mathrm{E}_{q(z_n|x_n;\theta)} \left[r(\theta, z_n, x_n) \right]$$

$$r(\theta, z_n, x_n) = \log \frac{p(x_n \mid z_n; \theta) p_0(z_n)}{q(z_n \mid x_n; \theta)}$$

Computing gradient:

$$\nabla_{\theta} \mathrm{E}_{q(z_n|x_n;\theta)} \left[r(\theta, z_n, x_n) \right]$$

Gradient of ELBO

$$r(\theta, z_n, x_n) = \log \frac{p(x_n \mid z_n; \theta) p_0(z_n)}{q(z_n \mid x_n; \theta)}$$

Computing gradient:

$$\nabla_\theta \mathrm{E}_{q(z_n \mid x_n; \theta)} \left[r(\theta, z_n, x_n) \right]$$

Gradient of ELBO

$$r(\theta, z_n, x_n) = \log \frac{p(x_n \mid z_n; \theta) p_0(z_n)}{q(z_n \mid x_n; \theta)}$$

Computing gradient:

$$\nabla_\theta \mathrm{E}_{q(z_n|x_n;\theta)} \left[r(\theta, z_n, x_n) \right] = \mathrm{E}_{q(z_n|x_n;\theta)} \left[\nabla_\theta r(\theta, z_n, x_n) \right] + \int r(\theta, z_n, x_n) \nabla_\theta q(z_n \mid x_n; \theta) d_{z_n}$$

1. sample $z_n \sim q(z_n \mid x_n; \theta) = \mathcal{N}(f(x_n)_1, f(x_n)_2)$, then compute average of $\nabla_\theta r(\theta, z_n, x_n)$

Gradient of ELBO

$$r(\theta, z_n, x_n) = \log \frac{p(x_n \mid z_n; \theta) p_0(z_n)}{q(z_n \mid x_n; \theta)}$$

Computing gradient:

$$\nabla_\theta \mathrm{E}_{q(z_n|x_n;\theta)} \left[r(\theta, z_n, x_n) \right] = \mathrm{E}_{q(z_n|x_n;\theta)} \left[\nabla_\theta r(\theta, z_n, x_n) \right] + \int r(\theta, z_n, x_n) \nabla_\theta q(z_n \mid x_n; \theta) d_{z_n}$$

2. rewrite as

$$\int r(\theta, z_n, x_n) \nabla_\theta q(z_n \mid x_n; \theta) d_{z_n} = \mathrm{E}_{q(z_n|x_n;\theta)} \left[r(\theta, z_n, x_n) \nabla_\theta \log q(z_n \mid x_n; \theta) \right]$$

then sample $z_n \sim q(z_n \mid x_n; \theta) = \mathcal{N}(f(x_n)_1, f(x_n)_2)$

compute average of $r(\theta, z_n, x_n) \nabla_\theta q(z_n \mid x_n; \theta)$

Problem — high variance

Reparameterization Trick

$$q(z_n | x_n; \theta) = \mathcal{N}(f(x_n)_1, f(x_n)_2) = \mathcal{N}(\mu_\theta(x_n), \Sigma_\theta(x_n))$$

Treating $\epsilon \sim N(0,1)$, standard Gaussian distribution, then

$$\mathrm{E}_{q(z_n|x_n;\theta)}\left[r(\theta, z_n, x_n)\right] = \mathrm{E}_{\epsilon \sim N(0,1)}\left[r(\theta, z_n, x_n)\right]$$

where $z_n = \Sigma_\theta^{\frac{1}{2}}(x_n)\epsilon + \mu_\theta(x_n)$

Taking gradient does not depend on the distribution

Reparameterization Trick

$$\nabla_\theta \mathrm{E}_{q(z_n|x_n;\theta)}\left[\log \frac{p(x_n \,|\, z_n;\theta)p_0(z_n)}{q(z_n \,|\, x_n;\theta)}\right]$$

$$= \nabla_\theta \mathrm{E}_{q(z_n|x_n;\theta)}\left[\log p(x_n \,|\, z_n;\theta)\right] - \mathrm{KL}\left(q(z_n \,|\, x_n;\phi) \| p_0(z_n)\right)$$

$$= \nabla_\theta \mathrm{E}_{\epsilon\sim N(0,1)}\left[\log p(x_n \,|\, z_n;\theta)\right] - \mathrm{KL}\left(\mathcal{N}(\mu_\theta(x_n), \Sigma_\theta(x_n)) \| \mathcal{N}(0,1)\right)$$

$$= \nabla_\theta \mathrm{E}_{\epsilon\sim N(0,1)}\left[\log p(x_n \,|\, z_n;\theta)\right] - \frac{1}{2}\left(\mu_\theta(x_n)^T\mu_\theta(x_n) + \mathrm{tr}(\Sigma_\theta)(x_n) - M - \mathrm{logDet}(\Sigma_\theta(x_n))\right)$$

$$= \mathrm{E}_{\epsilon\sim N(0,1)}\left[\nabla_\theta \log p(x_n \,|\, z_n;\theta)\right] - \nabla_\theta \frac{1}{2}\left(\mu_\theta(x_n)^T\mu_\theta(x_n) + \mathrm{tr}(\Sigma_\theta(x_n)) - M - \mathrm{logDet}(\Sigma_\theta(x_n))\right)$$

where $z_n = \Sigma_\theta^{\frac{1}{2}}(x_n)\epsilon + \mu_\theta(x_n)$

Compute Gradient using Reparameterization Trick

For each data point x_n, current parameter θ

Step 1: sample $\epsilon \sim N(0,1)$

Step 2: using encoder forward to compute $\mu, \Sigma = f_{\text{enc}}(x_n; \theta)$

Step 3: $z(\theta) = \Sigma^{\frac{1}{2}}\epsilon + \mu$

Step 4: using decoder forward to compute $p(x_n \mid z(\theta); \theta)$

Step 5: define

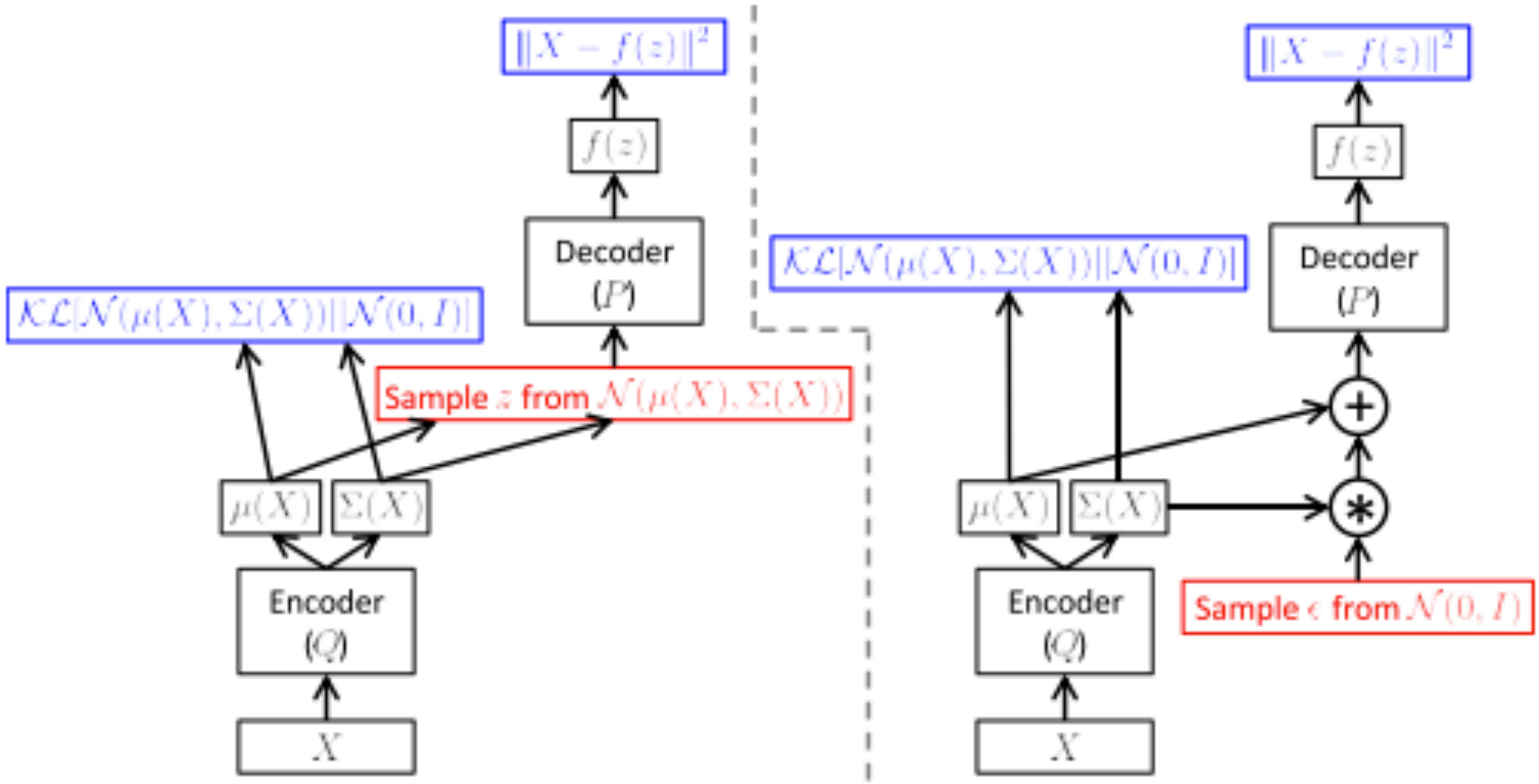
$\text{err} = \log p(x_n \mid z_n; \theta) - \beta \cdot \text{KL}\left(q(z \mid x_n; \theta) \| p_0(z)\right)$, then

using back-propagation to compute gradient for θ

$$\frac{1}{2}\left(\mu_\theta(x_n)^T \mu_\theta(x_n) + \text{tr}(\Sigma_\theta)(x_n) - M - \text{logDet}(\Sigma_\theta(x_n))\right)$$

Training VAE

- Reparameterization trick

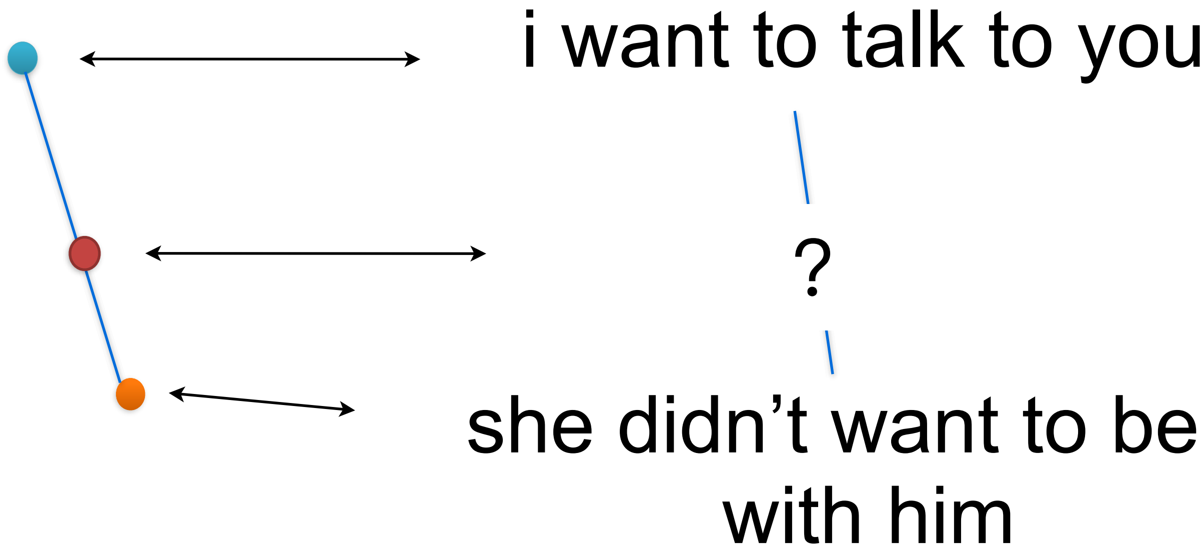


Tutorial on Variational Autoencoders
(Doersch Carl, 2016)

Sentence VAE

Generating Sentence from Continuous vectors

- Key challenge: Interpolation in continuous space should yield reasonable sentences



Conditional Sequence Generation

Given a latent variable z, a sequence of text tokens $x = (x_1, x_2, \ldots, x_t)$ can be generated with RNN (or LSTM, transformer), CRNN model:

$$p\left(x \mid z; \theta\right) = \prod_t p(x\_t \mid x_{<t}, z; \theta)$$

$$p\left(x_t \middle| x_{<t}, z; \theta\right) = \mathrm{softmax}(W \cdot h_t)$$

$$h_t = RNN(h_{t-1}, \left[x_{t-1},\ z\right], \theta)$$

VAE for Sentence Generation

Decoding:

$$z \sim N(0, I)$$

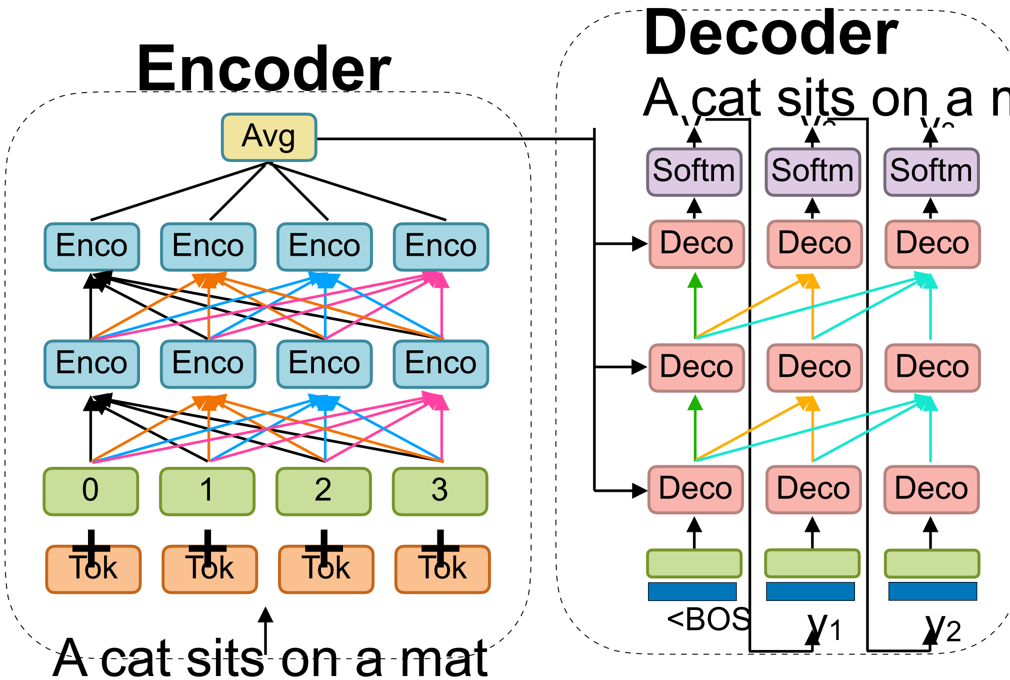
generate x from Transformer(z) or LSTM(z)

Encoding:

$$q(z \mid x) = N\left(\mu, \sigma^2\right)$$

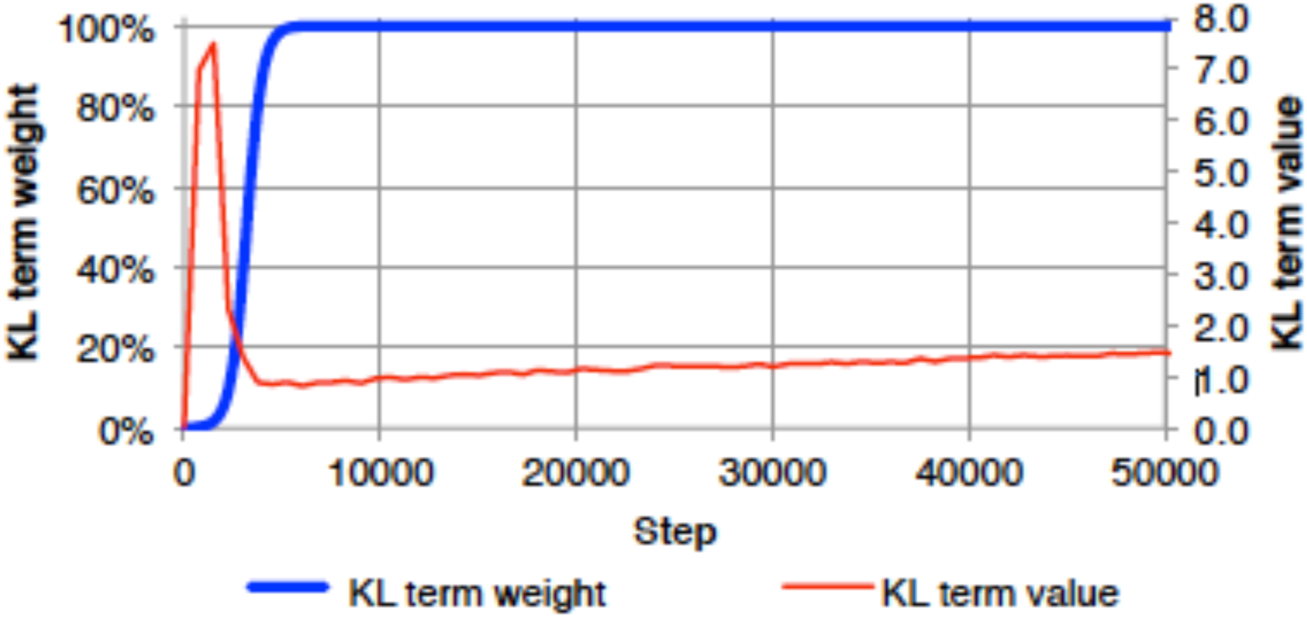
$$\mu = W_1 \cdot h_t, \ \sigma^2 = \exp W_2 \cdot h_t$$

$$h_t = \text{Transformer}(x; \theta)$$



Training VAE: Posterior Collapse

- KL term in ELBO collapses to zero and latent variable encodes little information.
- Solution: KL annealing & word dropout



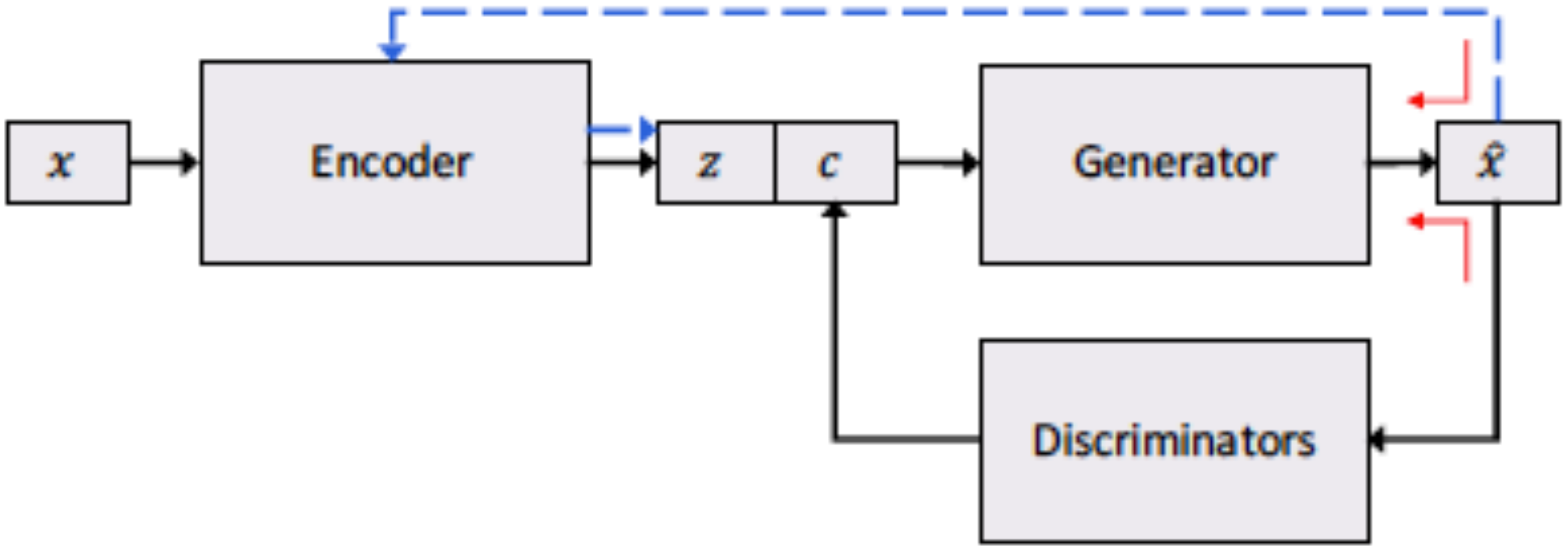
Examples on Sentence Interpolation

“ i want to talk to you . ”
“i want to be with you . ”
“i do n’t want to be with you . ”
i do n’t want to be with you .
she did n’t want to be with him .

he was silent for a long moment .
he was silent for a moment .
it was quiet for a moment .
it was dark and cold .
there was a pause .
it was my turn .

Variants

- Controllable sentence generation with both continuous and discrete labels



Toward Controlled Generation of Text, (Hu et. al. ICML 2017)

Generating with Varying Semantic Label

the film is strictly routine !
the film is full of imagination .

after watching this movie , i felt that disappointed .
after seeing this film , i 'm a fan .

the acting is uniformly bad either .
the performances are uniformly good .

this is just awful .
this is pure genius .

the acting is bad .
the movie is so much fun .

none of this is very original .
highly recommended viewing for its courage , and ideas .

too bland
highly watchable

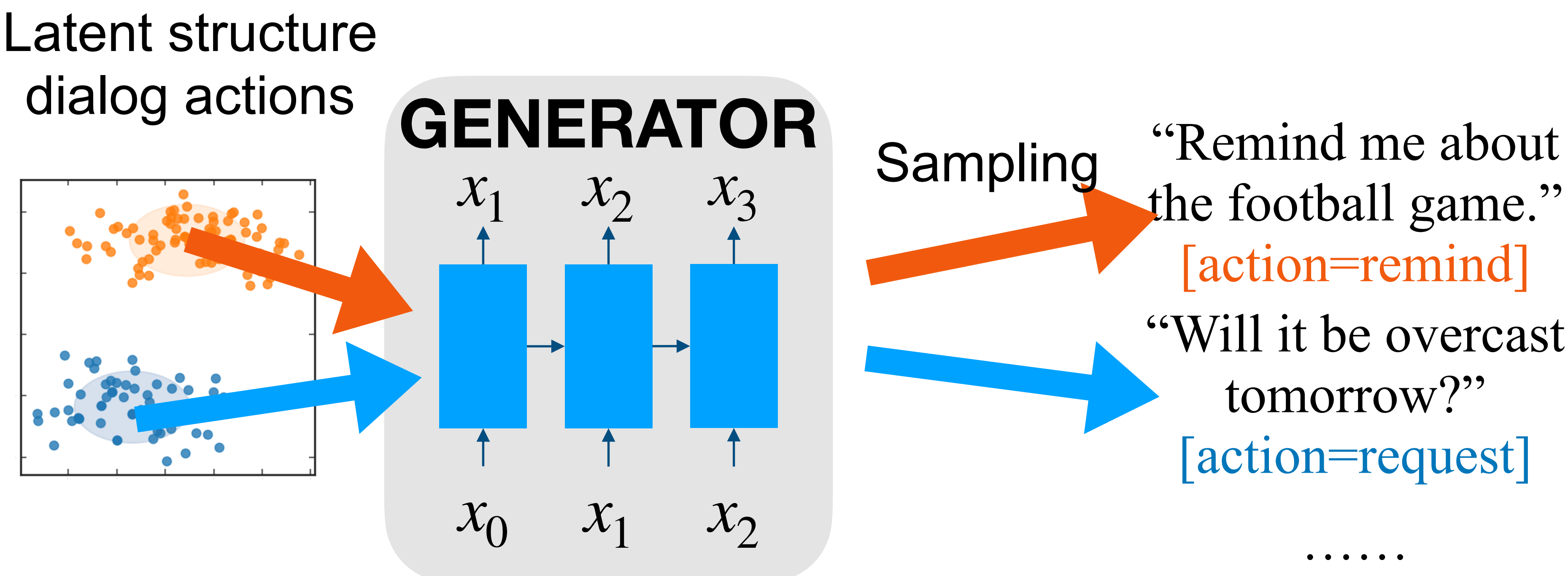
i can analyze this movie without more than three words .
i highly recommend this film to anyone who appreciates music .

Toward Controlled Generation of Text, (Hu et. al. ICML 2017)

Deep Latent Variable Models for Text

- Interpretable Deep Latent Representation from Raw Text
  - Learning Exponential Family Mixture VAE [ICML 20]
- Disentangled Representation Learning for Text Generation
  - Data to Generation: VTM [ICLR 20b]
  - Learning syntax-semantic representation [ACL 19c]
- One model to acquire 4 language skills
  - Mirror Generative NMT [ICLR 20a]

Learning Interpretable Latent Representation

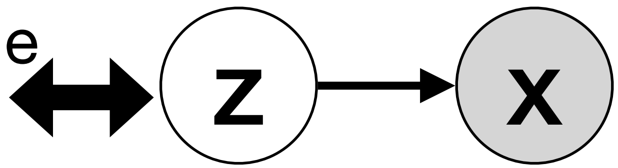


Generate Sentences with interpretable factors

How to Interpret Latent Variables in VAEs?

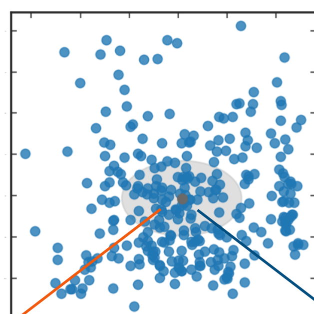
Variational Auto-encoder (VAE)

interpretable structure



(Kingma & Welling, 2013)

z: **continuous** latent variables



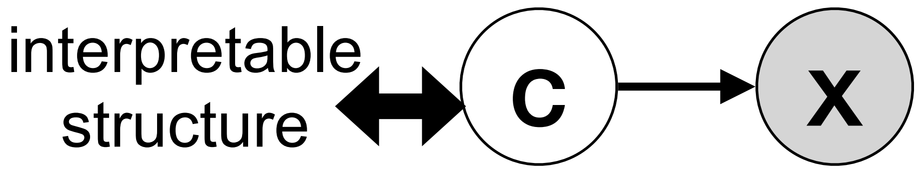
Will it be humid in New York today?

Remind me about my meeting.

difficult to interpret discrete factors

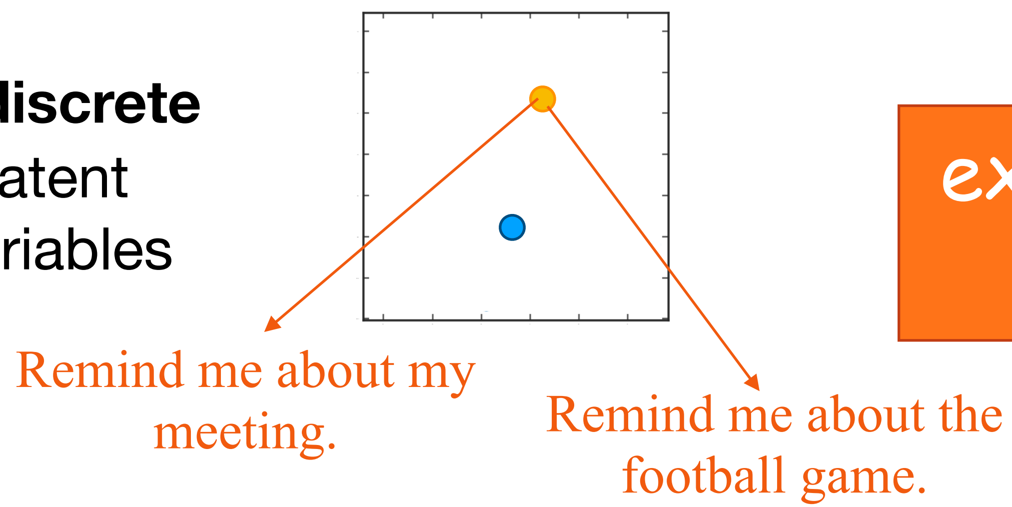
VAEs Introduce Latent Variables

Variational Auto-encoder (VAE)



(Zhao et al, 2018b)

c: **discrete** latent variables

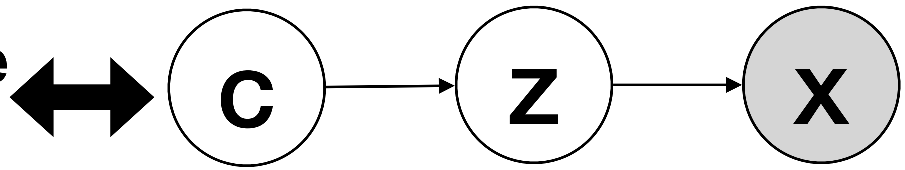


expressiveness is limited.

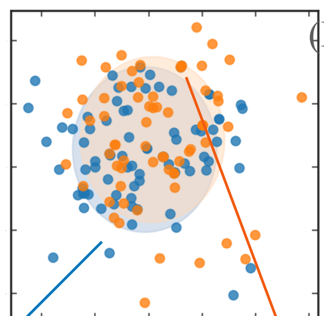
Discrete Variables Could Enhance Interpretability - but one has to do it right!

Gaussian Mixture Variational Auto-encoder (GM-VAE)

interpretable structure



(Dilokthanakul et al., 2016; Jiang et al., 2017)



c: **discrete** component

z: **continuous** latent variable

Will it be overcast tomorrow?

Remind me about the football game.

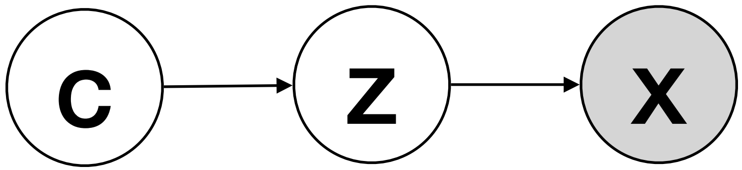
Why?

How to fix it?

mode-collapse

Do it right for VAE w/ hierarchical priors - Dispersed Exponential-family Mixture VAE

Exponential-family Mixture VAE



adding dispersion term in training

Dispersed EM-VAE

$$L(\theta; x) = \text{ELBO} + \beta \cdot L_d,$$

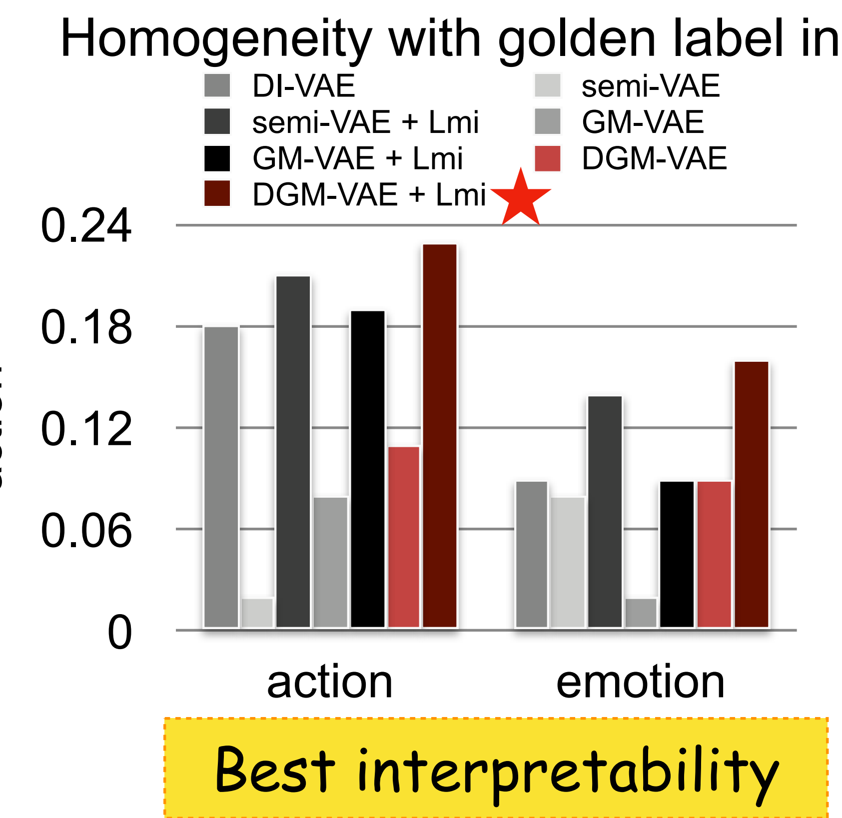
dispersion term

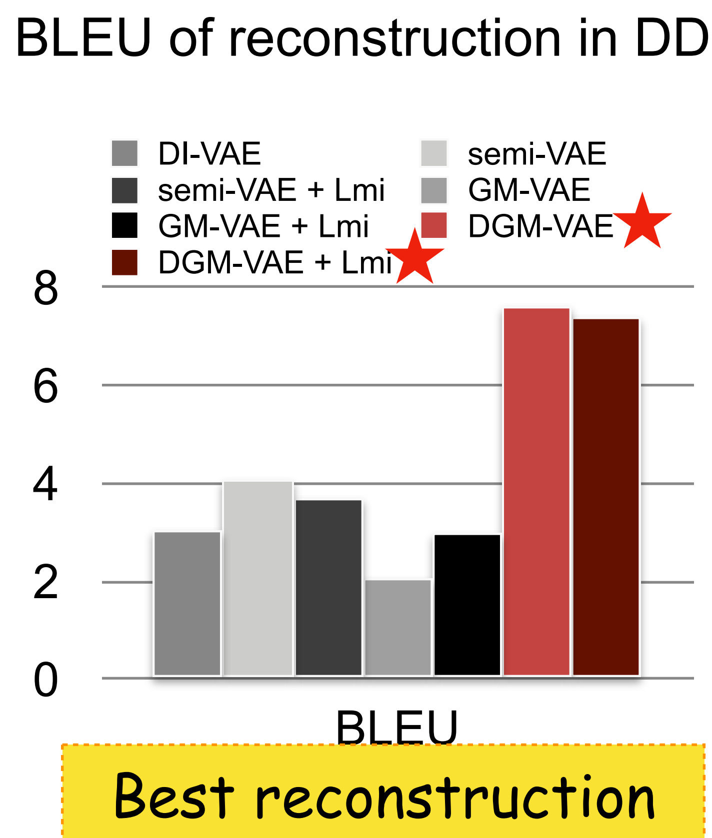
$$L_d = \mathbb{E}_{q_\phi(c|x)} A(\boldsymbol{\eta}_c) - A(\mathbb{E}_{q_\phi(c|x)} \boldsymbol{\eta}_c) \, .$$

DEM-VAE [W. Shi, H. Zhou, N. Miao, **Lei Li**, ICML 2020]

Generation Quality and Interpretability

DGM-VAE obtains the best performance in interpretability and reconstruction





DEM-VAE [W. Shi, H. Zhou, N. Miao, **Lei Li**, ICML 2020]

Latent Variables Learned by DEM-VAE are Semantically Meaningful

Example actions and corresponding utterances (classified by $q_\phi(c\,|\,x)$)

Inferred action=Inform-route/address

"There is a Safeway 4 miles away."
"There are no hospitals within 2 miles."
"There is Jing Jing and PF Changs."
…

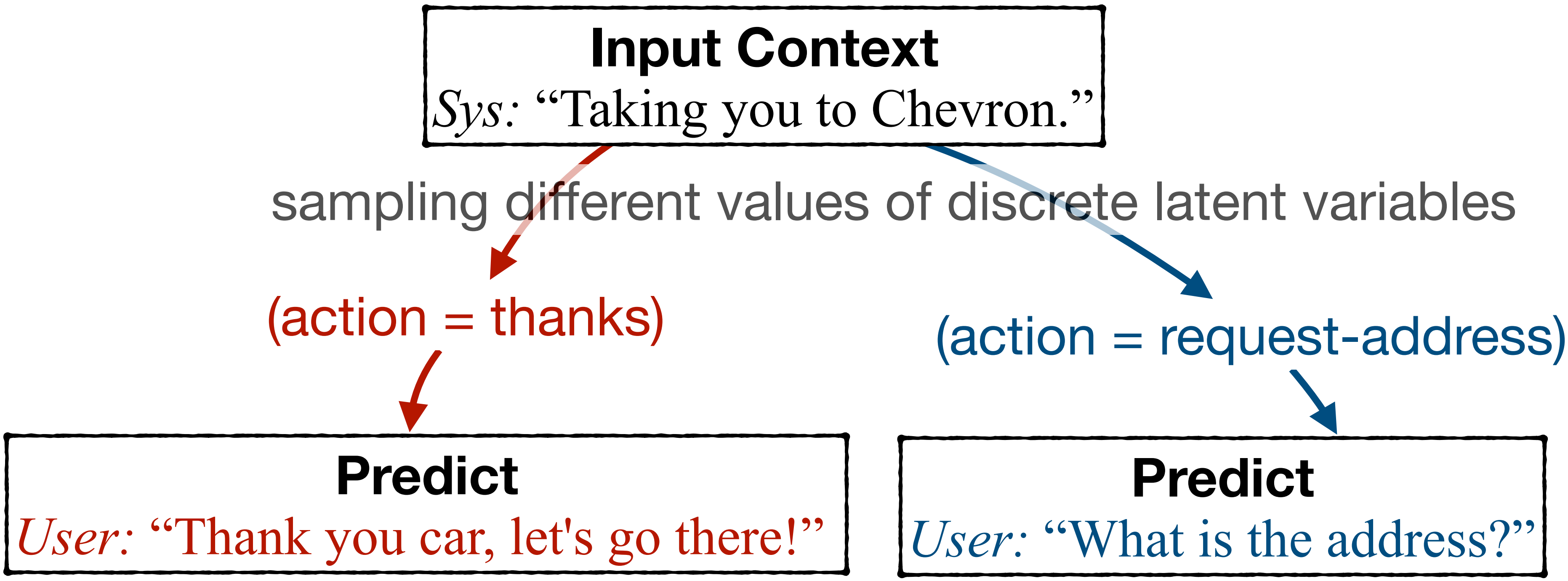
Inferred action =Request-weather

"What is the weather today?"
"What is the weather like in the city?"
"What's the weather forecast in New York?"
…

Utterances of the same actions could be assigned with the same discrete latent variable c.

DEM-VAE [W. Shi, H. Zhou, N. Miao, **Lei Li**, ICML 2020]

Generate Sensible Dialog Response with DEM-VAE



Responses with different actions are generated by sampling different values of discrete latent variables.

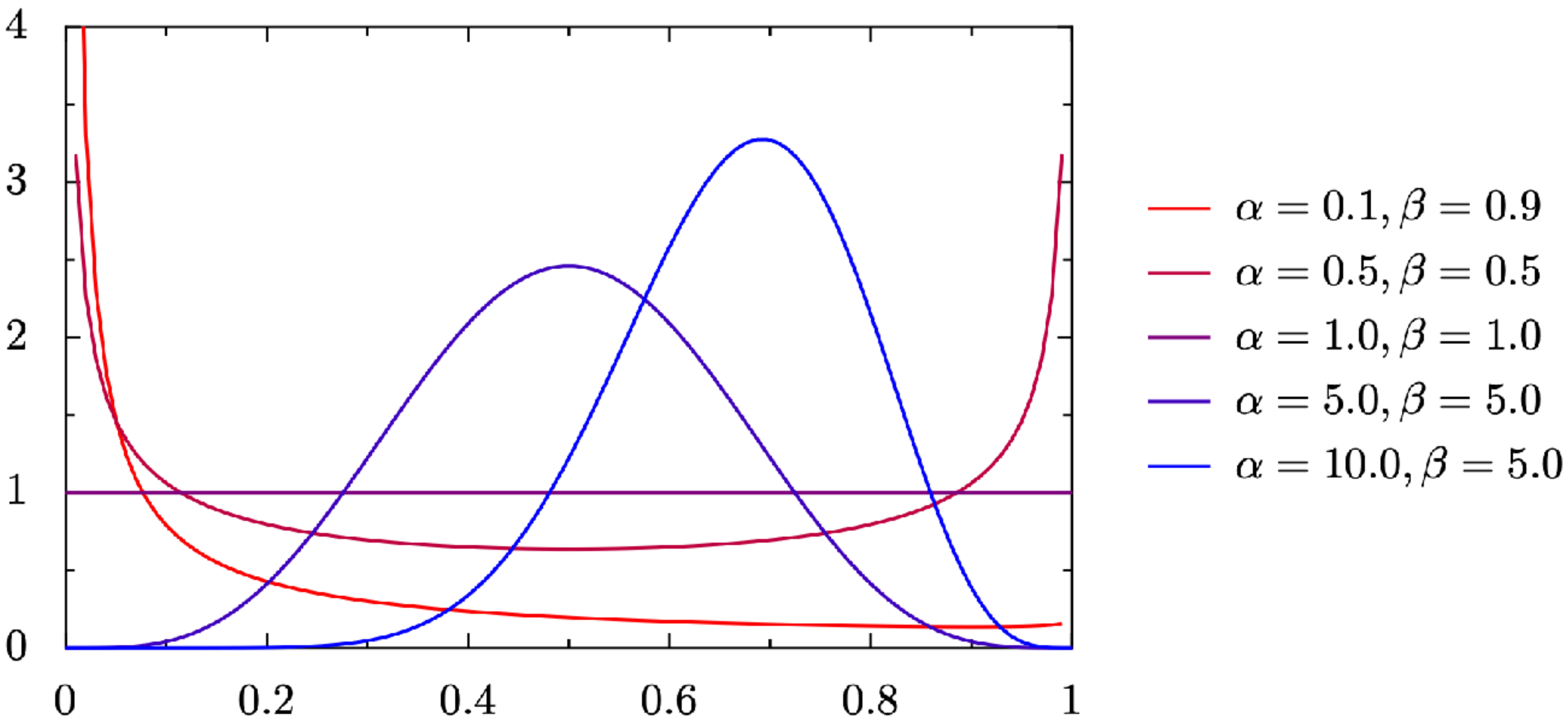
DEM-VAE [W. Shi, H. Zhou, N. Miao, **Lei Li**, ICML 2020]

Topic Modelling

- We want to automatically find themes/grouped keywords from a collection of articles
  - e.g. finding the trending topics from NYT news of past 100 years
  - finding scientific topics from all papers published in Science/ Nature/PNAS
- Tell the covered topics of each article, and proportion change over time
- discover latent topics from corpus

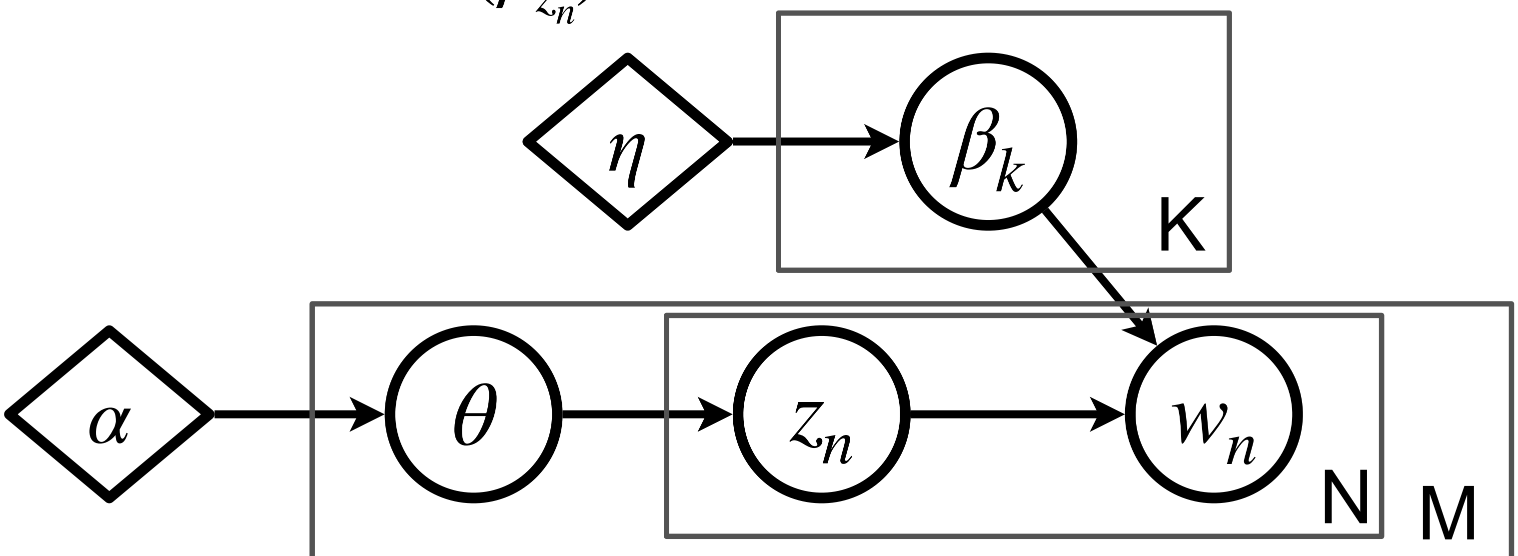
Dirichlet Distribution

$$p(\theta \mid \alpha) = \frac{1}{B(\alpha, \beta)} \prod_{k=1}^{K} \theta_k^{\alpha_k - 1}, \text{ where } B(\alpha) = \frac{\prod_{k=1}^{K} \Gamma(\alpha_k)}{\Gamma(\prod_{k=1}^{K} \alpha_k)}$$



Latent Dirichlet Allocation

- K topics, M docs, each with N words
- $\theta \sim Dir(\alpha)$
- $z_n \sim Multinomial(\theta)$
- $w_n \sim Multinomial(\beta_{z_n})$



$$p(w) = \sum_z \int p(\theta)p(\beta)(\prod_{n=1}^{N} p(z_n \mid \theta)p(w_n \mid \beta_{z_n}))d\theta d\beta$$

Latent Dirichlet Allocation

- A generative model for document
- Each word is generated from a topic's distribution over vocabulary
- A document is a mixture of proportions (topic vector)
- Also known as Mixed membership models

Inference and Learning for LDA

- Inference:
  - given a document D
  - estimate $P(\theta \mid \alpha, \beta, D)$
- Learning:
  - given a collection of documents $\{D_m\}$
  - Estimate parameters α, β

$$\arg\max \sum_m \log P(D_m \mid \alpha, \beta)$$

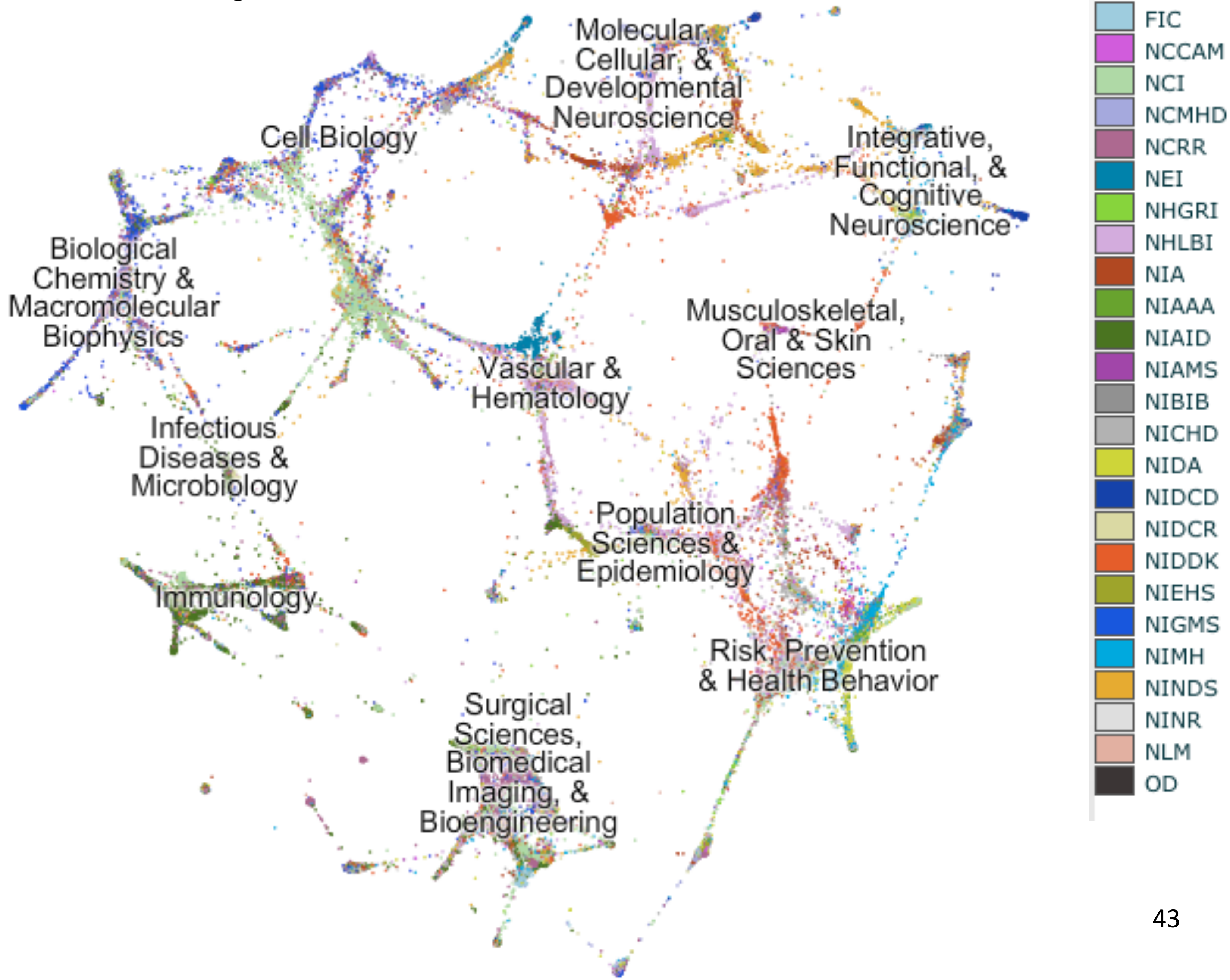
Approximate Inference

- Variational inference
  - using a variational distribution (fully factorized) to approximate posterior
- MCMC
  - Gibbs sampling

"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
MOVIE	BILLION	YEARS	TEACHERS
PLAY	FEDERAL	FAMILIES	HIGH
MUSICAL	YEAR	WORK	PUBLIC
BEST	SPENDING	PARENTS	TEACHER
ACTOR	NEW	SAYS	BENNETT
FIRST	STATE	FAMILY	MANIGAT
YORK	PLAN	WELFARE	NAMPHY
OPERA	MONEY	MEN	STATE
THEATER	PROGRAMS	PERCENT	PRESIDENT
ACTRESS	GOVERNMENT	CARE	ELEMENTARY
LOVE	CONGRESS	LIFE	HAITI

The William Randolph Hearst Foundation will give $1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be $200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive $400,000 each. The Juilliard School, where music and the performing arts are taught, will get $250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual $100,000 donation, too.

Topics of NIH grants



Summary

- Auto-Encoder: learning representation by reconstruction
- Variational Auto-Encoder: put prior on latent representation and use variational method to train
- Variational method is a general approximation method for intractable density

Next Up

- Monte Carlo sampling