1. Let \( M = (Q, \Sigma, \delta, q_0, F) \) be a deterministic finite acceptor or dfa, where
the symbols express the usual notation (refer page 38 of the text). We
define the generalized transition function \( \delta^* : Q \times \Sigma^* \rightarrow Q \) as

(a) \( \delta^*(q, \lambda) = q \)
(b) \( \delta^*(q, wa) = \delta(\delta^*(q, w), a) \)

for all \( q \in Q, w \in \Sigma^* \) and \( a \in \Sigma \).

What is the language \( L(M) \) accepted by the above dfa in terms of the
generalised transition function? Can you express \( \overline{L(M)} \)?

\[ \text{Figure 1: example DFA} \]

2. What is the language \( L \) accepted by the DFA in Figure 1.

3. Show that if we change Figure 1, making \( q_3 \) as the non final state and
making \( q_0, q_1, q_2 \) as the final states, the resulting dfa accepts \( L \). Can you
generalise (Page 47, #4)?

4. For \( \Sigma = \{a, b\} \), construct dfa’s that accept the sets consisting of
(a) all strings with exactly one \( a \).
(b) all strings with at least one \( a \).
(c) all strings with at least one \( a \) and exactly two \( b \)'s.

5. Find dfa’s for the following languages on \( \Sigma = \{a, b\} \).
(a) \( L_{a,k} = \{a^n : n \geq k\} \); for \( k = 4 \).
(b) \( L = \{ab^5wb^2 : w \in \{a, b\}^*\} \).
(c) \( L_{a,2} \cup L_{b,2} \).

Hint: Think about running DFAs for each language concurrently.