



# A Markov-Based Channel Model Algorithm for Wireless Networks

ALMUDENA KONRAD\*, BEN Y. ZHAO and ANTHONY D. JOSEPH  
*Computer Science Division, UC Berkeley, Berkeley, CA, USA*

REINER LUDWIG  
*Ericsson Research, Herzogenrath, Germany*

**Abstract.** Techniques for modeling and simulating channel conditions play an essential role in understanding network protocol and application behavior. In [11], we demonstrated that inaccurate modeling using a traditional analytical model yielded suboptimal error control protocol parameters choices. In this paper, we demonstrate that time-varying effects on wireless channels result in wireless traces which exhibit non-stationary behavior over small window sizes. We then present an algorithm that extracts stationary components from a collected trace in order to provide analytical channel models that, relative to traditional approaches, more accurately represent characteristics such as burstiness, statistical distribution of errors, and packet loss processes. Our algorithm also generates artificial traces with the same statistical characteristics as actual collected network traces. For validation, we develop a channel model for the circuit-switched data service in GSM and show that it: (1) more closely approximates GSM channel characteristics than traditional Markov models and (2) generates artificial traces that closely match collected traces' statistics. Using these traces in a simulator environment enables future protocol and application testing under different controlled and repeatable conditions.

**Keywords:** modeling, wireless channel, Markov model, error statistics, GSM

## 1. Introduction

As networks evolve, the design of communication protocols increases in complexity. Evaluating the performance of existing networks provides insights into techniques for optimizing future protocols. The most common techniques include simulation, analysis of empirical data, and analytical models (e.g., channel models). Accurate modeling of network events, especially the error behavior at link layer and above, is essential to the understanding of network behavior and to the design of communication protocols. For example, a detailed understanding of the packet loss process and burstiness of errors is necessary for the proper design and parameter tuning of error control protocols, such as Automatic Repeat reQuest (ARQ) protocols.

Streaming audio and video multimedia applications can also benefit from a better understanding of the underlying network behavior. For example, video and audio codecs can perform real-time predictive rate control by using a model of network traffic characteristics to estimate traffic conditions in real-time.

The traditional network modeling approach to error modeling is to create a Gilbert model [17] (i.e., a two-state discrete time Markov model) based upon collected network traffic traces. Using this model, one can then dynamically generate artificial network traces for the network under study and use the traces to simulate, and thus, better understand the performance of existing and new network protocols and appli-

cations. These traces provide network protocol and application developers with ease of use and repeatability, two critical characteristics for developing and improving network and application performance. More importantly, for new networks under development (or for which there are only limited prototypes), it is often difficult to collect a reasonable amount of traces or to run experiments. By generating synthetic traces that simulate the network being tested, multiple users can simultaneously gain network access and perform experiments.

Unfortunately, as we will show, Markov models have several significant shortcomings in the accuracy of their error modeling, which directly affect the validity of results based upon traces generated from these models. Models based upon Markov processes require that the error statistics remain relatively constant over time. Many networks experience time varying effects, such as congestion-related losses. Wireless channels, in particular, experience effects such as Rayleigh fading, multipath fading, shadowing, etc. While previous work has not focused on the stationarity of traces, we hypothesize that wireless traces exhibit non-stationary behavior over small window sizes, and that by isolating and analyzing stationary trace segments, more accurate models can be developed. Utilizing a previously published, but not widely known algorithm for testing stationarity [2], we tested 215 min of wireless traces and confirmed its non-stationarity with a derived window size. This implies that traditional stochastic analysis of wireless traces are likely to be less accurate than ideal.

Thus, we propose and evaluate a novel algorithm, the *Markov-based Trace Analysis* (MTA) algorithm, for the de-

\* Corresponding author.  
E-mail: almudena@cs.berkeley.edu

sign of channel error models. Our approach is to derive a statistical constant from the wireless trace, and use this constant to identify lossy and error-free segments of transmission. From the original trace we extract a stationary subtrace representing lossy segments of transmission. By analyzing the length distributions of the lossy and error-free segments, we can effectively characterize the transitions between them, and create a model that more accurately represents the original trace.

In practice, this MTA algorithm allows a more accurate analysis of network traces which accounts for their non-stationary behavior. This characteristic makes MTA a general purpose algorithm, meaning that it can be applied to network traces such as wireless traces which experience different error statistics over time. However, the purpose of this work is not to show that the MTA algorithm is general purpose, but to argue that the MTA algorithm generates accurate analytical models for wireless channels.

We validate the benefits and accuracy of the MTA algorithm by applying it to 215 min of GSM digital wireless cellular network [15] data traces collected at the reliable link layer (Radio Link Protocol layer [5,7]) to generate a model we call the *MTA GSM channel model*. We then show that, unlike traces generated by Markov models, artificial MTA model network traces have the same statistical properties as traces collected from the actual network. Such traces will provide more accurate simulations of the network being tested, yielding results that more closely match the results obtained on actual networks.

In particular, we generate artificial traces using the MTA, Gilbert, and third-order Markov models, and perform *retrace analysis* [11] on these artificial traces. *Retrace analysis* emulates an enhanced RLP layer using a fixed data frame size and fixed per frame overhead (e.g., checksums, sequence numbers, etc.), and calculates the predicted throughput over a range of fixed RLP frames sizes. In our enhanced RLP implementation, frame sizes are multiples of the physical radio block size of 30 bytes.<sup>1</sup> For a given frame size, there is a tradeoff between the increased throughput from reducing overhead and the retransmission delay caused when a radio block of an RLP frame is lost and the entire frame is retransmitted. In other words, a greater frame size leads to (1) lower overhead, and (2) longer retransmission delay (more radio blocks have to be retransmitted) when a radio block is corrupted. Thus, throughput performance results for each frame size are highly correlated with a collected or synthetic trace's error statistics. In [11], we used *retrace analysis* to show that for bursty error traces (where errors tend to occur in clusters), larger frames yield higher throughput. Furthermore, we showed that incorrectly assuming an even distribution of errors in GSM leads to the wrong choice of optimal frame size.

These results show that the distribution of errors within traces has a significant influence on models, analysis, and simulations based upon such traces. This conclusion is es-

pecially true when the goal is to artificially generate traces for the design, simulation, and analysis of new networking protocols. To replicate and further explore the results from our earlier work, we generate an artificial trace that we call *even error distribution (EED) trace*, which has the same error rate as collected traces, but with an even error distribution (i.e., errors are individual events, isolated, and have a constant distance between each other).

The rest of this paper is organized as follows. We start by discussing related work in the next section. Section 3 provides background information about GSM's Circuit-Switched Data (CSD) service and an overview on Discrete Time Markov Chains. Next, in section 4, we describe our measurement platform for collecting frame level error traces on the GSM wireless link. Then section 5 shows the development of the MTA algorithm, followed by section 6, where we develop three analytical models for GSM wireless traffic: the MTA model, the Gilbert model, and the third-order Markov model. In section 7, we present our algorithm for generating artificial traces and evaluate the MTA algorithm by comparing the traffic statistics of the collected and artificial traces. We conclude and discuss our plans for future work in section 8.

## 2. Related work

Several researchers have explored ways of characterizing the loss process of various channels. Bolot et al. [3] use a characterization of the loss process of audio packets to determine an appropriate error control scheme for streaming audio. They model the loss process as a two-state Markov chain, and show that the loss burst distribution is approximately geometric. Yajnik et al. [20] characterize the packet loss in a multicast network by examining the spatial (across receivers) and temporal (across consecutive packets) correlation in packet loss. Of particular interest is their modeling of temporal loss as a third-order Markov chain. Both these efforts analyze the loss process of traces with static error statistics (i.e., the error rates do not vary over time). However, our work addresses the additional challenge of modeling traces with time-varying error statistics.

There is also interesting related work in wireless traffic modeling. Nguyen et al. [16] use a trace-based approach for modeling wireless errors. They present a two-state Markov wireless error model, and develop an improved model based on collected WaveLAN error traces. Building on this, Balakrishnan and Katz [1] also collected error traces from a WaveLAN network and developed a two-state Markov chain error model (i.e., Gilbert model). Zorzi et al. [21] also investigate the error characteristics in a wireless channel. They compare an independent and identically distributed (IID) model to the Gilbert model, and claim that higher order models are not necessary. Their results are drawn by applying these models to artificial traces generated by assigning a fixed-average burst length and a constant bit error rate.

While these previous works confirm that Markov models improve upon the simple IID model, we offer proof in this

<sup>1</sup> Note that the existing GSM RLP implementation uses a frame size of one radio block.

paper that these models have several significant shortcomings in their error modeling accuracy. Furthermore, we argue that there is a need to develop a more accurate model based on real world statistics that better describes and handles time-varying wireless channel error characteristics. Previous work such as that done by Yajnik et al. modeled loss processes using higher-order Markov chains for improved accuracy, but was limited to stationary traces. In this work, we provide an algorithm that successfully models non-stationary error traces.

### 3. Background

In this section we present a brief background on the technology behind circuit-switched data in GSM networks. We also define Discrete Time Markov Chains (DTMC) and some of their relevant properties.

#### 3.1. Circuit-switched data in GSM

The Global System for Mobility (GSM) wireless digital cellular network is a second generation cellular network, providing nearly 700 million subscribers with global roaming capabilities in several hundred countries. GSM implements several error control techniques, including adaptive power control, frequency hopping, Forward Error Correction (FEC), and interleaving. The primary uses of the GSM network are for Circuit-Switched Voice service (CSV) and Short Message Service (SMS). However, an increasing number of subscribers are using GSM's Circuit-Switched Data service (CSD), which provides an optional reliable link layer protocol, the Radio Link Protocol (RLP). We provide a brief summary below; more details about GSM, the CSD service, and RLP can be found in [15].

GSM is a TDMA-based (Time Division Multiple Access) circuit-switched network. At call-setup time, a mobile terminal is assigned a user data channel, defined as the tuple (carrier frequency number, slot number). The slot cycle time is 5 ms on average. This timing allows 114 bits to be transmitted in each slot, yielding a gross data rate of 22.8 Kbit/s. The fundamental transmission unit in GSM is a *radio data block*. A Forward Error Correction (FEC) radio data block is 456 bits, representing the payload of 4 time slots. In GSM-CSD, the size of an unencoded data block is 240 bits, resulting in a raw data rate of 12 Kbit/s (240 bits every 20 ms) [6].

Interleaving is a technique that is used in combination with FEC to combat burst bit errors. Instead of transmitting a data block in four consecutive slots, the block is divided into smaller fragments. Fragments from different data blocks are then interleaved before transmission. The interleaving scheme chosen for GSM-CSD interleaves a single data block over 22 TDMA slots [8]. A few of these smaller fragments can be completely corrupted while the corresponding data block can still be reconstructed by the FEC decoder. The primary disadvantage of this deep interleaving is that it intro-

duces a significant one-way latency of approximately 90 ms.<sup>2</sup> This high latency can have a significant adverse effect on interactive protocols [12].

RLP [5,7] is a full-duplex logical link layer protocol that uses selective reject and checkpointing for error recovery. The RLP frame size is fixed at 240 bits aligned to the above mentioned radio data block. RLP introduces an overhead of 48 bits per RLP frame, yielding a user data rate of 9.6 Kbit/s in the ideal case (no retransmissions).<sup>3</sup> RLP transports user data as a transparent byte stream (i.e., RLP does not "know" about IP packets). However, RLP may lose data if a link reset occurs (e.g., after a maximum number of retransmissions of a single frame has been reached).

#### 3.2. Discrete time Markov chains

A Discrete Time Markov Chain (DTMC) [17] is a random process  $\{X_n \mid n \geq 0\}$  that takes values in a discrete space  $E$ . A DTMC is defined by its memory and its transition probabilities and is characterized as follows:

$$\begin{aligned} \Pr(X_{n+1} = j \mid X_0 = i_0, X_1 = i_1, \dots, X_n = i_n) \\ = \Pr(X_{n+1} = j \mid X_{n-z+1}, 1 \leq z \leq K), \end{aligned} \quad (1)$$

where  $\Pr(X_{n+1} = j \mid X_{n-z+1}, 1 \leq z \leq K)$  are the  $K_{\text{step}}$  transition probabilities, and  $K$  defines the memory.

To calculate the memory of a DTMC, we find the *order* of the Markov chain as first proposed in [14]. To aid in determining the order of the Markov chain, we introduce the concept of *conditional entropy*. The *conditional entropy* is an indication of the randomness of the next element of a trace, given the past history. We determine the amount of past history necessary by calculating the *i*th *order entropy* for  $1 \leq i \leq M$ , where  $M$  is an upper bound on the maximum amount of history we want to record. We choose  $M$  to be 6 because maintaining history for  $2^6$  or 64 states yields a reasonable level of implementation and processing complexity (more states implies higher computational time). An *i*th *order entropy* of 0 indicates that knowing the last *i* elements of the chain totally predicts the next element on the chain. As the entropy value increases, there is more randomness in the next element on the chain. We follow the same procedure used by Yajnik et al. [20] to calculate the conditional entropy for each value of *i*:

$$H(i) = - \sum_{\vec{x}} \frac{\xi(\vec{x})}{T_{\text{samples}}} \sum_{y \in \{0,1\}} \frac{\xi(y, \vec{x})}{\xi(\vec{x})} \log_2 \frac{\xi(y, \vec{x})}{\xi(\vec{x})}. \quad (2)$$

In equation (2),  $\vec{x}$  represents the vector  $[x_1, \dots, x_i]$  which corresponds to one of the  $2^i$  different patterns of *i* consecutive elements in the chain;  $T_{\text{samples}}$  represents the total number of samples of length *i* in the chain;  $\xi(\vec{x})$  indicates the number of times the pattern  $\vec{x} = [x_1, \dots, x_i]$  shows up in the chain;

<sup>2</sup> Note that voice is treated differently in GSM. Unencoded voice data blocks have a size of 260 bits and the interleaving depth is 8 slots.

<sup>3</sup> Note that the transparent (without RLP) GSM-CSD service introduces a wasteful overhead for modem control information, reducing the user data rate to 9.6 Kbit/s.

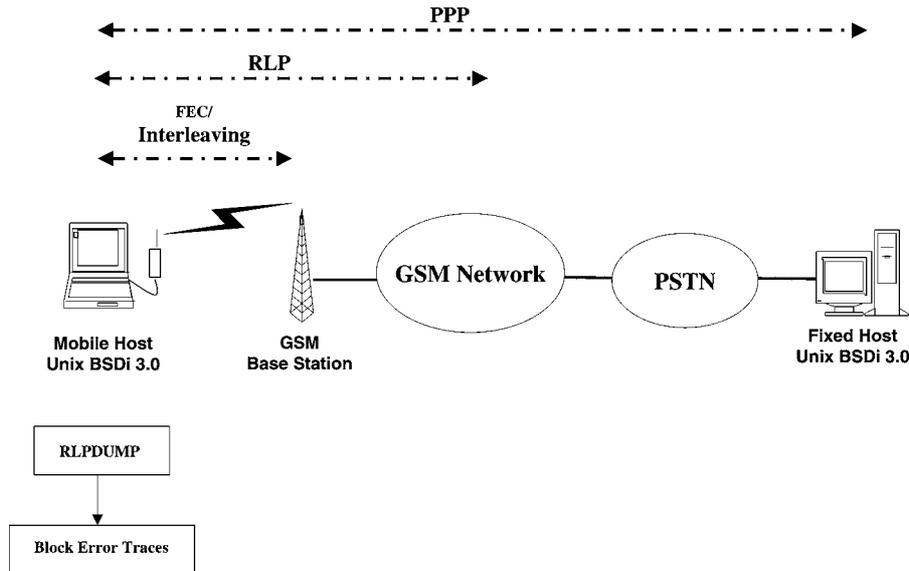


Figure 1. The GSM network and measurement platform.

and the term  $\xi(y, \vec{x})$  corresponds to the number of times the pattern  $\vec{x} = [x_1, \dots, x_i]$  appears in the chain followed by  $y$ , where  $y \in \{0, 1\}$ .

Given the implicit tradeoff between entropy and complexity of the Markov model, we choose the order of the Markov chain  $K$ , such that we gain the minimum entropy possible at an acceptable complexity level. As entropy decreases, the order  $K$  increases, meaning the number of states (i.e.,  $2^k$ ) increases exponentially.

#### 4. Data collection

In this section, we first introduce the concept of *frame error traces*. Then we describe the measurement platform we developed to collect these traces.

##### 4.1. Frame error traces

An accurate representation of a wireless channel's error characteristics for a given time period can be captured by a bit error trace. A bit error trace contains information about whether a particular bit was transmitted correctly (i.e., a "1" represents a corrupted bit, while a "0" represents a correctly transmitted bit). The average Bit Error Rate (BER) is the first-order metric commonly used to describe such a trace. The same approach can be applied on a frame level instead of on a bit level. A frame error trace consists of a binary sequence where each element represents the transmission state of a data frame. There are two frame states, a "1" represents a corrupted data frame, while a "0" represents a correct data frame. Corrupted frames are detected using an error detection code (e.g., Cyclic Redundancy Check). In this paper, we refer to frame error traces simply as traces. We also use the *Frame Error Rate* (FER) of a trace to define the average rate of corrupted data frames. For a trace, we define an *error burst* to be a run of

consecutive 1's, and an *error-free burst* as a run of consecutive 0's.

We have collected traces under several different scenarios. As shown in figure 1, we vary the movement of the mobile host (fixed, walking, and driving) while keeping the other endpoint fixed. We collected 215 min of traces in a fixed environment, where the mobile host's signal strength was below 4 on a scale of 1 to 5. In the following sections, we refer to this trace as the *GSM trace*. In section 6, we use the GSM trace to develop an analytical traffic model for RLP. Note that the error characteristics we have measured in these traces are only valid for the particular FEC and interleaving scheme implemented in GSM's Circuit Switched Data network (see section 3.1). To analyze other types of network channels, the first step is to collect frame or packet level traces and then to apply the analysis described below.

##### 4.2. Measurement platform

We depict our measurement platform in figure 1. A single-hop network running the Point-to-Point Protocol (PPP) [18] connects the mobile host to a fixed host that terminates the circuit-switched GSM connection. We used the *sock* tool [19] to generate traffic on the link. To collect traffic traces at the RLP layer, we ported the RLP protocol implementation of a commercial available GSM data PC-Card to BSDi3.0 UNIX. In addition, we developed RLPDUMP, a protocol monitor tool for RLP. RLPDUMP logs whether or not a received frame could be correctly recovered by the FEC decoder. This determination is possible because every RLP frame corresponds to an FEC encoded radio block (see section 3.1). Thus, a received block suffers an error whenever the corresponding RLP frame has a frame checksum error. We used *sock* to generate bulk data traffic and used RLPDUMP to capture frame error traces.

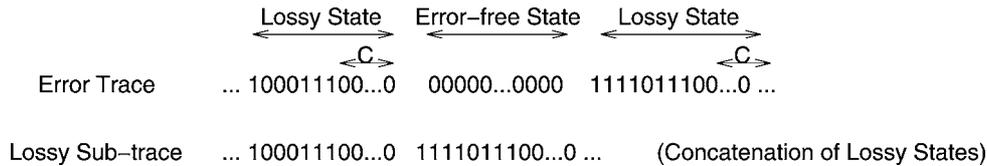


Figure 2. The separation of an error trace into two stationary traces.

## 5. The MTA algorithm

The basic concept behind the MTA algorithm is the assumption that an error trace with non-stationary error properties can be decomposed into a stationary subset consisting of what we define as *lossy states*. The MTA algorithm defines two states, the *lossy* and the *error-free states* and parameterizes transitions between them as a function of a preset parameter, the *change-of-state constant*.

Error-free states contain only correctly transmitted frames, while lossy states contains both error-free and error frames. We show that lossy states exhibit stationarity in its error statistics, and that they can be modeled by a traditional DTMC. The MTA algorithm computes the distribution of lengths for both error-free and lossy states, along with the memory and parameters for the DTMC used on the sequence of lossy states.

In this section, we first discuss stationarity properties and how to test a trace for stationarity. We then present the MTA algorithm and show how it is applied to a trace.

### 5.1. Stationarity

We consider a network traffic trace to be a random process  $\{X_n \mid n \geq 0\}$  with a discrete space  $E = \{0, 1\}$ , where a 1 denotes a corrupted frame and a 0 denotes a correct transmitted frame. If  $X_n = i$ , then the process is said to have value  $i$  at time  $n$ . A process  $X_n$  that takes values on the discrete space  $E = \{0, 1\}$  is also called a binary time series [4]. One major challenge in the analysis of time series is the concept of *stationarity*. A process  $X_n$  is strictly stationary if the distribution of  $(X_{p+1}, \dots, X_{p+k})$  is the same as that of  $(X_1, \dots, X_k)$  for each  $p$  and  $k$ .  $X_n$  is second-order stationary if the mean  $m_n = E(X_n)$  is constant (independent of  $n$ ), and the autocovariance only depends on the difference  $k$  for all  $n$  ( $\text{Cov}(k, n) = \text{Cov}(X_n, X_{n-k}) = \text{Cov}(k)$ ). Given a binary time series  $X_n$  that is second-order stationary, the process can be modeled as a DTMC where the value of the chain at time  $n$  is determined by the memory of the process [10]. However, checking a trace for stationarity is mathematically challenging.

We define a trace to be *stationary* whenever the error statistics remain relatively constant over time. This definition depends on the window size we are using to examine the trace. Figure 3 shows that GSM trace consists of error and error-free bursts, where the length of error-free bursts are significantly longer than the length of error bursts. In other words, the traces consist of long error-free segments interrupted by small error clusters [13]. Note that for channels with relatively small error clusters, examining traces using a large window size value not only lowers the perceived channel error

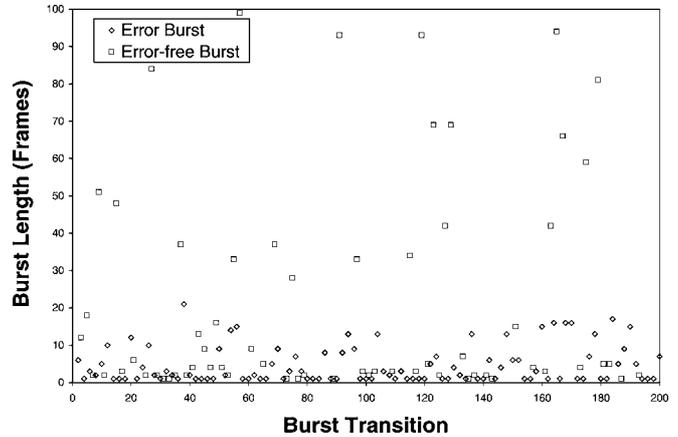


Figure 3. Burst length in GSM trace.

rate, but also distorts the statistics needed by DTMCs, resulting in less accurate models. As the window size decreases towards the length of the average error burst, the channel exhibits significantly different error characteristics.

We identify trace sections that exhibit stationary properties by finding error-free bursts of length equal to or greater than the change-of-state constant  $C$ . The value of  $C$  is a design decision that we define as the mean plus one standard deviation of the length of error bursts of a trace. By removing trace sections consisting of error-free bursts of length equal to or greater than  $C$ , we guarantee that the resulting trace will have stationarity error statistic properties. We explain the reasoning behind our choice in more detail in section 6.1. We next define a lossy state as a sequence of zeros and ones (always started by a one), where each run of zeros is not greater than the change-of-state constant  $C$ . To test for stationarity in wireless traces we need to choose a window size close to the average size of the lossy state.

We use the test for stationarity introduced by Bendat and Piersol called the Runs Test [2], summarized as follows:

1. Define a run as a number of consecutive ones (also referred to as an error burst).
2. Divide the trace into segments of equal lengths.
3. Compute the lengths of runs in each segment.
4. Count the number of runs of length above and below the median value for run lengths in the trace.
5. Plot a histogram for the number of runs.

For a stationary trace, the number of runs distribution between the 0.05 and 0.95 cutoffs will be close to 90% [2].

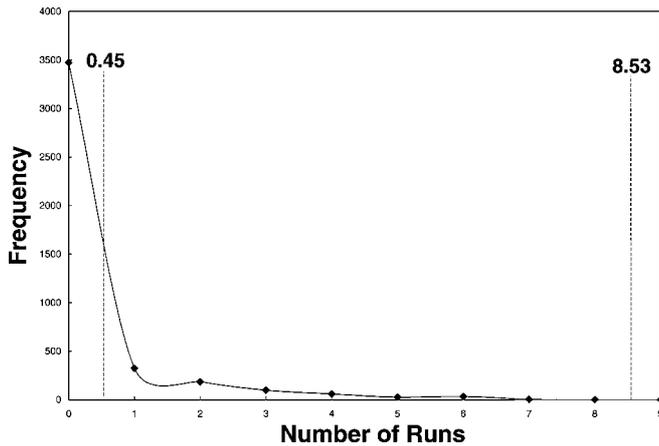


Figure 4. The Runs Test applied to GSM trace.

We apply the Runs Test to test GSM trace for stationarity. We first calculate the mean and standard deviation for the error burst length. In this case, the mean value was found to be 6 frames and the standard deviation was 14 frames, yielding a state-of-change constant value  $C$  of 20 ( $6 + 14$ ) frames. The average error cluster size was found to be 26 frames and the standard deviation was 54 frames. We choose the window size for the Runs Test to be 50.

Figure 4 shows that only 17% of the runs distribution lie between the 0.05 and 0.95 cutoffs, and 83% lays outside the left and right cutoffs. Thus, from the Runs Test, we conclude that GSM trace is a non-stationary process for a window size of 50. In the following sections we use the term stationarity to refer to stationarity for window size of 50.

## 5.2. Algorithm

The MTA algorithm views a trace as a process with two types of states: lossy and error-free. The algorithm extracts lossy states from the error trace and concatenates these states to form *lossy subtrace* (as defined in section 5.1) (see figure 2).

We define two random processes with a discrete space  $E = \{0, 1, 2, \dots\}$ :

- The *lossy state length* process  $\{B_n \mid n \geq 0\}$ , where  $B_n$  represents the number of elements in the  $n$ th lossy state (i.e., the length of the state).
- The *error-free state length* process  $\{G_n \mid n \geq 0\}$ , where  $G_n$  represents the length of the  $n$ th error-free state.

The distributions of  $B_n$  and  $G_n$  are found by plotting the cumulative density function (CDF) and finding the “best” fitting distributions. We provide an example of how to determine these distributions in section 7.1.

The lossy subtrace is a stationary random process, therefore, it can be modeled as a DTMC with a certain memory. The MTA algorithm calculates the memory of the lossy subtrace, and determines its transition probabilities.

The application of the MTA algorithm to an input trace can be summarized as follows:

1. Calculate the mean ( $m_e$ ) and standard deviation ( $sd_e$ ) values for error burst lengths in the trace.

2. Set  $C$ , the change-of-state constant, equal to  $(m_e + sd_e)$ .
3. Partition the trace into lossy state and error-free state portions using the following definitions:
  - *Lossy state*: runs of 1’s and 0’s, with the first element being a 1, and with runs of 0’s that have length less than or equal to the  $C$ .
  - *Error-free state*: runs of 0’s that have length greater than  $C$ .
4. Create lossy subtrace from the lossy state portions of the error trace.
5. Model lossy subtrace as a DTMC, and calculate its order and transition probabilities.
6. Determine the best fitting distributions of the length processes  $B_n$  and  $G_n$ .

In summary, to take advantage of the Markov process properties in non-stationary traces, we have used a novel approach to error modeling: a *Markov-based Trace Analysis* (MTA) algorithm that extracts from an error trace a subset trace that have stationary properties.

## 6. Modeling GSM wireless channel

In this section, we demonstrate the process of extracting characteristic statistics from a given trace using the MTA and Markov models [17]. We apply all three algorithms to GSM trace to generate the statistics which we will later use to generate artificial traces based on each model.

### 6.1. MTA GSM model

This section presents an application of the steps of the MTA algorithm (as described in section 5) to GSM trace.

First, the MTA algorithm analyzes the *error-free* and *error* burstiness experienced by GSM trace (see figure 3), and calculates the state-of-change constant value  $C$ . Section 5.1 calculated  $C$  to be 20. Since our goal is to isolate and analyze sections that experience stationarity, we use the MTA algorithm to create a stationary subtrace, called *lossy subtrace*. The MTA algorithm creates this subtrace (as described in section 5) by first identifying lossy states and then concatenating these states to form lossy subtrace. Figure 5 shows the error-free bursts and error burstiness experienced by lossy subtrace. In this plot, the average error free burst is 3.26 frames, with a maximum value of 20 frames (recall that the change-of-state constant  $C$  was defined to be 20). The error free burst mean and maximum values in lossy subtrace are much smaller than the error burst mean and maximum value in GSM trace. Thus, our choice of  $C$  guarantees that lossy subtrace will experience constant error statistic properties and therefore stationarity. To prove that lossy subtrace is an stationary process we apply the Runs Test. Figure 7 shows that 87% of the runs distribution lie between the 0.05 and 0.95 cutoffs. Therefore, this

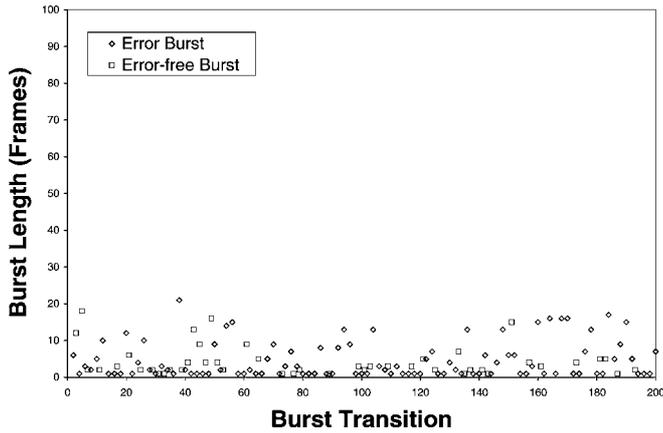


Figure 5. Burst length in lossy subtrace.

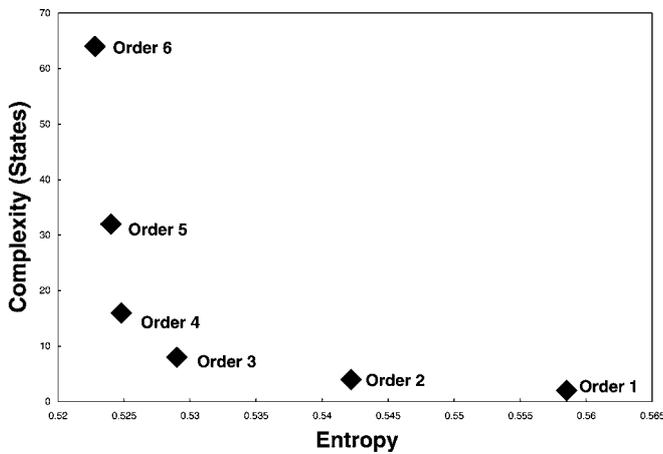


Figure 6. Complexity versus entropy in lossy subtrace.

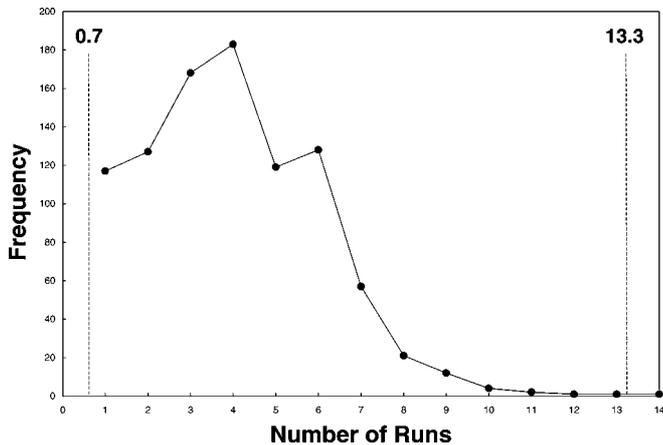


Figure 7. The Runs Test applied to lossy subtrace.

result proves that lossy subtrace is a stationary process and can thus be modeled as a DTMC.

Next, the MTA algorithm models lossy subtrace as a DTMC with memory  $K$ . To determine the memory  $K$  of the DTMC, the MTA algorithm first calculates the conditional entropy values. Table 1 shows the conditional entropy calculated for different  $K$  values. Figure 6 illustrates how the complexity of the DTMC measured in number of states increases

Table 1  
Entropy for the lossy subtrace.

Order $K$	Entropy
6	0.5228
5	0.5240
4	0.5248
3	0.5290
2	0.5422
1	0.5585

Table 2  
Fourth-order Markov model statistics.

State $i$	$\Pr(i)$	$\Pr(1   i)$	$\Pr(0   i)$
0000	0.1254	0.1699	0.8301
0001	0.0305	0.6414	0.3586
0010	0.0172	0.1832	0.8168
0011	0.0344	0.8009	0.1991
0100	0.0166	0.3073	0.6927
0101	0.0033	0.8129	0.1871
0110	0.0087	0.2683	0.7317
0111	0.0415	0.8889	0.1111
1000	0.0305	0.3022	0.6978
1001	0.0210	0.7037	0.2963
1010	0.0027	0.0547	0.9453
1011	0.0159	0.8820	0.1180
1100	0.0350	0.4556	0.5444
1101	0.0153	0.8623	0.1377
1110	0.0415	0.3118	0.6882
1111	0.5604	0.9341	0.0659

exponentially as entropy decreases. For this trace we chose  $K$  to be 4 (i.e., 16 number of states), which corresponds to only 0.38% increase in entropy from the chosen upper bound of  $K = 6$ . We could have chosen  $K$  to be larger than 4, but we did not want to significantly increase the complexity of the Markov model.

Table 2 shows the probabilities of the trace being in each state and the associated transition probabilities. The transition probabilities were also calculated by frequency counting.

The last step of the MTA algorithm is to determine the best fitting distribution for the lossy state length process  $B_n$  and error-free state length process  $G_n$ . Figures 8 and 9 show the CDF for the processes  $B_n$  and  $G_n$ . Each figure shows two plots, one plot is the CDF as calculated from the empirical data (i.e., the distribution of GSM trace), and the other plot corresponds to the CDF of an exponential distribution with parameter  $\alpha$ . We assume that the distributions of  $B_n$  and  $G_n$  are exponential with parameter  $\alpha$  (i.e., the CDF  $F(x) = 1 - e^{-\alpha x}$ , where  $x$  is the error-free or lossy state length). For each distribution,  $B_n$  and  $G_n$ , the MTA algorithm plots the CDF of the exponential distribution with  $\alpha$  ranging from 0 to 1 in steps of 0.001, and then chooses a value of  $\alpha$  that provides the best approximation to the empirical data's CDF (i.e., the distribution for GSM trace). We denote  $\vec{x}$  as the vector with the CDF values based on the empirical data, and  $\vec{y}$  as the vector with the CDF values based on the predicted exponential distribution. We use the *standard error* as a measure of the error between plots, and choose the dis-

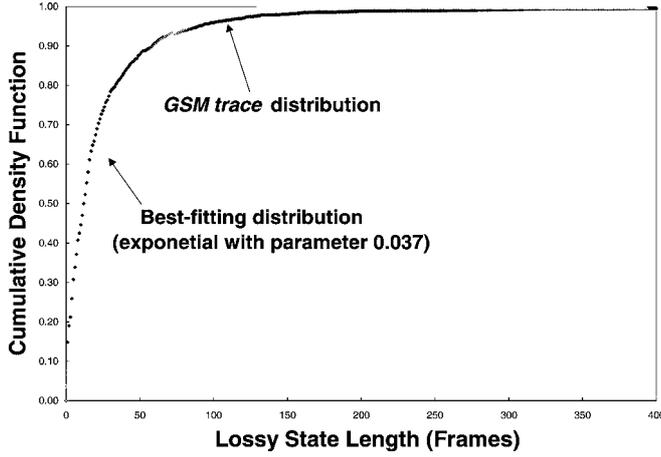


Figure 8. Lossy state length distribution.

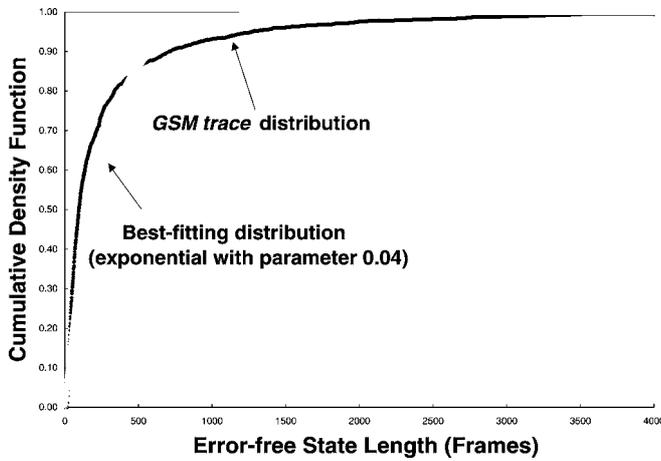


Figure 9. Error-free state length distribution.

tribution with smallest standard error. The equation for the standard error of the predicted  $\vec{y}$  is

$$S_{\text{error}}(\vec{y}, \vec{x}) = \sqrt{\left[ \frac{1}{n(n-2)} \right] \left[ f(y) - \frac{[n \sum xy - \sum x \sum y]^2}{f(x)} \right]}, \quad (3)$$

where  $f(a) = n \sum a^2 - (\sum a)^2$ , and  $n$  is the dimension of the vectors  $\vec{y}$  and  $\vec{x}$ .

The predicted distributions for the lossy and error-free state lengths are exponential distributions with parameters  $\alpha_b = 0.037$  and  $\alpha_g = 0.04$ , respectively. The standard error values for the predicted distributions of  $B_n$  and  $G_n$  are 0.013 and 0.025, respectively. Note that a lower standard error value indicates a more accurate prediction.

## 6.2. The Markov GSM models

To study the performance and accuracy of the MTA algorithm, we compared the MTA model to two Markov-chain based models, the traditional Gilbert model and a third-order Markov model. The Gilbert model is a DTMC of order one (i.e., with two states). In our traces, the Gilbert model states

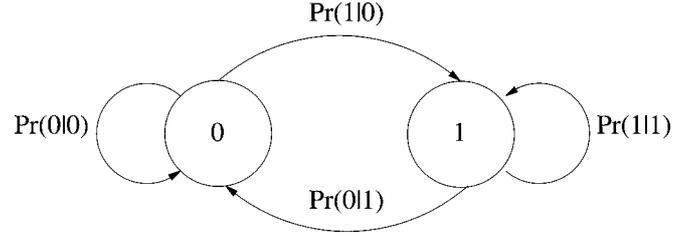


Figure 10. Gilbert model state transition diagram.

Table 3  
Gilbert model statistics.

State $i$	$\text{Pr}(i)$	$\text{Pr}(1   i)$	$\text{Pr}(0   i)$
0	0.9449	0.0087	0.9913
1	0.0551	0.8509	0.1491

Table 4  
Third-order Markov model statistics.

State $i$	$\text{Pr}(i)$	$\text{Pr}(1   i)$	$\text{Pr}(0   i)$
0 0 0	0.9299	0.0056	0.9944
0 0 1	0.0068	0.5488	0.4512
0 1 0	0.0034	0.1140	0.8860
0 1 1	0.0048	0.8087	0.1913
1 0 0	0.0068	0.2360	0.7640
1 0 1	0.0014	0.7654	0.2346
1 1 0	0.0048	0.2139	0.7861
1 1 1	0.0421	0.9074	0.0925

correspond to the status of each data frame  $\{0, 1\}$ , where a 1 denotes a corrupted frame and a 0 denotes a correct frame. The Gilbert model predicts the state of the next frame by just looking at the previous received frame. Figure 10 shows the Gilbert model state transition diagram. Table 3 shows the results of the Gilbert model transition probability calculations for GSM trace.

The third-order Markov model is a DTMC of order three (i.e., with eight states). Compared to Gilbert, this model keeps track of the status of the previous three frames, increasing its prediction accuracy at the cost of additional complexity in the Markov chain. Table 4 shows the transition probability calculations for the third-order Markov model for GSM trace.

## 7. Trace generation and evaluation

A key capability of the MTA algorithm is the ability to generate artificial traces (of any duration) with the same statistical characteristics as traces collected from any given network. In this section, we demonstrate how to generate an artificial trace given characteristic statistics from the MTA model. We also generate two artificial traces based on the Gilbert and third-order Markov models, and compare all three artificial traces against the GSM trace. We show that with respect to key characteristics such as error burst length distribution and throughput versus frame size, the MTA artificial trace provides a much improved approximation of the original GSM trace.

### 7.1. MTA artificial trace generation

The algorithm for trace generation from an MTA model is as follows:

1. Choose the number of frames,  $N$ , to generate in the artificial trace.
2. The algorithm repeats the following steps until all  $N$  frames have been generated:
  - (a) Determine  $g_{\text{len}}$ , the error-free state length from the error-free state length distribution  $G_n$ .
  - (b) Determine  $b_{\text{len}}$ , the lossy state length from the lossy state length distribution  $B_n$ .
  - (c) Generate  $g_{\text{len}}$  error-free frames (i.e., a sequence of "0" of length  $g_{\text{len}}$ ).
  - (d) Generate  $b_{\text{len}}$  frames that are either error or error-free frames depending on the transition probabilities calculated for the *lossy subtrace* in the MTA model.

Recall that in the MTA model, we observed that the lossy and error-free state length distributions,  $B_n$  and  $G_n$ , fit exponential distributions. Thus, to calculate  $b_{\text{len}}$  and  $g_{\text{len}}$  we can use the inverse transformation method from [9]. Given a random variable  $X$  with a CDF  $F(x)$ , the variable  $u$  is uniformly distributed between 0 and 1. We can generate a sample value of  $X$  by generating  $u$  and calculating  $x = F^{-1}(u)$ . In the exponential case with parameter  $\alpha$ ,  $u = F(x) = 1 - e^{-\alpha x}$ ,  $x$  can be determined from  $x = -\ln u/\alpha$ . In each case,  $x$  corresponds to either  $g_{\text{len}}$  or  $b_{\text{len}}$ .

It should be clear by inspection that an artificial trace created by the above algorithm is guaranteed to have the same characteristics as those extracted by the MTA algorithm.

### 7.2. Trace comparison

Here we evaluate the MTA algorithm by comparing the error statistics of the GSM trace against the three artificial traces. Figure 11 plots each CDF for the error burst lengths of the four traces. The mean, standard deviation, and maximum values are summarized in table 5. Note that GSM trace and the MTA artificial trace experience similar burst characteristics with 95% of the error burst lengths being smaller than 22 frames long, while in the Gilbert trace 95% of the error burst lengths are of size 1, and in the third-order Markov trace 95 are of size 4. These results show that the error burst distribution of the MTA trace represents a much closer approximation to the collected trace, GSM trace.

To demonstrate the importance of an accurate model for setting system parameters, we cite an example where a naive assumption about the channel statistics can lead to poor performance. In [11], we showed how an inaccurate channel model can lead to poor decision on the optimal RLP frame size of an enhanced multiple radio block implementation (see section 1). We repeat this demonstration using the GSM trace, artificial traces from MTA, Gilbert, third-order Markov, and an artificial trace based on trivial assumptions we call *even error distribution (EED) trace*. We artificially generated EED

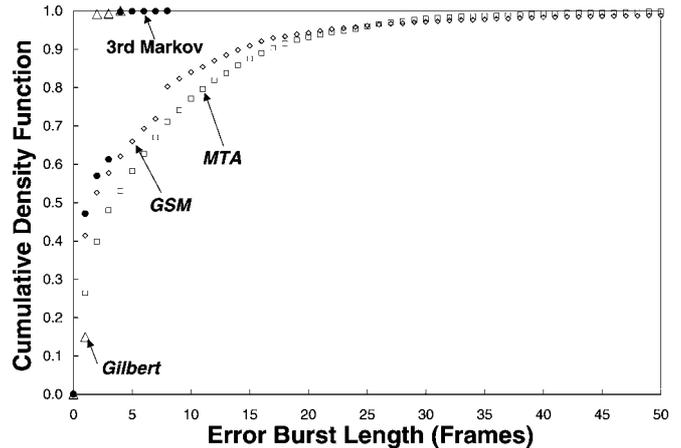


Figure 11. Error burst length distribution.

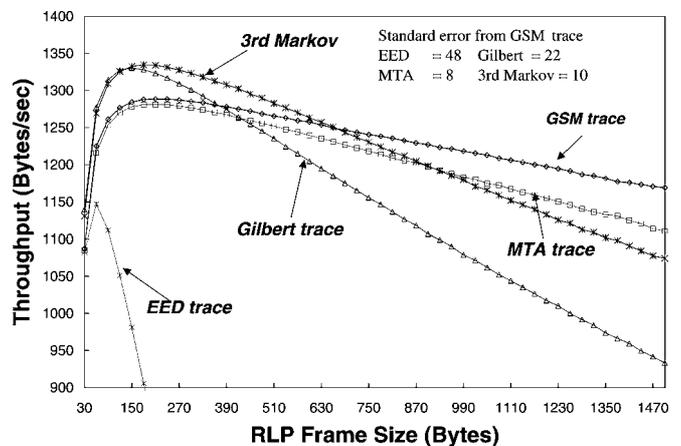


Figure 12. Retrace analysis of four traces.

Table 5  
Error length statistics.

Trace	Mean	Standard deviation	Maximum
GSM trace	6	14	126
MTA trace	7.0	8.1	82
Gilbert trace	1.8	0.4	4
Third-order Markov trace	2.35	0.02	8

trace with the same FER as GSM trace, but with an even error distribution. We then perform *retrace analysis* on the four artificial traces, yielding the results shown in figure 12. Note that the throughput for EED trace decreases dramatically as frame size increases, yielding an optimal frame size of only 60 bytes or 2 radio blocks. The Gilbert trace experiences higher throughput values for small frame sizes, but throughput decreases rapidly as the frame size increases. Its optimal frame size is 150 bytes (5 radio blocks). The third-order Markov trace has an optimal frame size of 180 bytes, which improves upon the Gilbert estimate. In contrast, the throughput plots for GSM trace and the MTA trace follow similar paths. Furthermore, they both yield an optimal frame size of 210 bytes (7 radio blocks). In this particular case, retrace analysis shows that the improved accuracy of the MTA

artificial trace over the Markov artificial traces leads to a more optimal design decision.

We used the standard error equation (see equation (3)) to measure how closely each artificial trace approximates the GSM trace. The standard error for EED trace was 48, for Gilbert trace was 22, for third-order Markov trace was 10, and for MTA trace was 8. Small standard error values signify that the traces experience more similar error statistics.

In summary, we used the characteristics from the MTA, Gilbert, and third-order Markov models to generate artificial traces, and used these traces to measure how accurately these algorithms model real traces. Both CDF and retrace analysis show that the artificial trace from the MTA model more accurately portrays the original GSM trace. Thus, we conclude that the MTA model provides a more accurate approximation technique than the traditional Markov models.

## 8. Conclusion

In this paper, we present a novel algorithm for modeling networks channels that experience time varying error statistics. The time varying nature of wireless and some wired channels has been a limiting factor in the analysis or modeling using Discrete Time Markov Chains. However, our *Markov-based Trace Analysis* algorithm and techniques allow us to separate a non-stationary network trace into stationary traces and to accurately model the traces using DTMCs.

We compare the application of the MTA model, the traditional Gilbert model and the third-order Markov model to traces collected in the GSM wireless digital cellular networks, and show that MTA model synthetic traces have burst error distributions that are closer to the real distributions of collected traces than the distribution of traces generated from the Gilbert and third-order Markov models.

We further show that when using *retrace analysis* to calculate the throughput for different frame sizes, our MTA model yields the correct optimal frame size decision, whereas less accurate models including the Gilbert model, the third-order Markov model, and an even error distribution model yield incorrect and non-optimal frame sizes. The results of the retrace analysis gives an example where a less accurate traffic model leads to the wrong design decision.

We are in the process of applying the MTA model to the problem of modeling the third generation GSM networks, including the General Packet Radio Service (GPRS). These networks currently have limited prototype deployment, making experimentation difficult. However, by creating MTA models for each network, we will enable easy, rapid experimentation and prototyping.

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**Almudena Konrad** is in her final year of completing her dissertation research for her doctoral degree in computer science from the University of California at Berkeley. She received her B.S. and M.S. degrees in electrical engineering at the University of California at Berkeley in 1997 and 1999. She was a National Science Foundation recipient from 1997 to 2000. Her areas of research are in the area of computer networks with an emphasis in the modeling, and analysis of network measurements.

E-mail: almudena@cs.berkeley.edu



**Ben Y. Zhao** is a Ph.D. candidate in the computer science division of the Department of Electrical Engineering and Computer Sciences at UC Berkeley. He received his B.S. with distinction and honors in computer science from Yale University in 1997, and his M.S. in computer science from Berkeley in 2000. His dissertation work is on the Tapestry wide-area location and routing infrastructure, and his research interests include large-scale distributed systems, their algorithms, data structures, applications and wireless

and mobile systems.

E-mail: ravenben@cs.berkeley.edu



**Reiner Ludwig** received his diploma and doctoral degrees in computer science from the University of Technology in Aachen, Germany, in 1994 and 2000. He pursues his research as an employee of Ericsson Research where he joined in 1994. He has spent over two years, between 1997 and 1999, at the University of California at Berkeley, working as a guest researcher on behalf of Ericsson Research. He currently focuses his research on the performance analysis of end-to-end protocols, and their interaction with

wireless link and physical layer functions.

E-mail: Reiner.Ludwig@eed.ericsson.se



**Anthony Joseph** received his Ph.D. degree from MIT in 1998, and joined the faculty of UC Berkeley in the Department of Electrical Engineering and Computer Sciences. His primary research interests are in mobile and distributed computing, wireless communications (networking and telephony), and smart spaces. He is exploring these areas in two efforts, the Iceberg project and the Ninja project, and in a broader collaboration, the Internet-scale Systems Research Group.

E-mail: adj@cs.berkeley.edu