CS 130A: Review I **Winter 2019**

1. Answer True or False for each of the following:

   (a) $O(n) + O(\log n) + O(\sqrt{n}) \;=\; O(n)$.

   (b) $\sum_{i=1}^{\sqrt{n}} i \;=\; \Theta(n)$.

2. We use a random hash function to map n keys into a table of size m, where m is an even integer. What is the probability that no key hashes into an **odd-numbered** location?

3. Prove Markov's inequality. If X is a non-negative random variable, and $a > 0$, then $Pr(X \geq a) \;\leq\; E(X)/a$.

4. Show the following regarding the maximum key in a binary min-heap with n nodes.

   (a) It must be at one of the leaves.

   (b) There are $\lceil n/2 \rceil$ leaves.

   (c) Any algorithm to find the maximum must take $\Theta(n)$ time.

5. Suppose we have a d-heap with N keys initially. We perform M PERCOLATEUP and N PERCOLATEDOWN operations on it.

   (a) What is the total running time for all these operations in terms of N, M and d?

   (b) What is the running time if $d = 2$.

   (c) What is the running time if $d = \Theta(N)$?

   (d) What choice of d achieves the minimum total running time?