Typeset your answers. TAs will not grade assignments with poor handwriting.
Staple any loose sheets. Do not ask me for a stapler in class.

1. (15 pts) Let $A_1, \ldots, A_6$ be six matrices, where $A_1$ is $10 \times 20$, $A_2$ is $20 \times 5$, $A_3$ is $5 \times 40$, $A_4$ is $40 \times 2$, $A_5$ is $2 \times 10$, and $A_6$ is $10 \times 10$. Determine the optimal way to compute their product $A_1 A_2 A_3 A_4 A_5 A_6$. Show the dynamic programming table, with all the subproblem solutions.

2. (15 pts) Let $A$, $B$, and $C$ be three strings each $n$ characters long. We want to compute the longest subsequence that is common to all three strings. Give a polynomial time algorithm for this problem. Derive the dynamic programming recurrence, clearly explain what each term stands for, and justify the correctness of the recurrence. Explain how you design an efficient algorithm using this recurrence, and analyze its time complexity.

3. (15 pts) Let $\mathcal{U}$ be a finite set, and let $S = \{S_1, S_2, \ldots, S_m\}$ be a collection of subsets of $\mathcal{U}$. Given an integer $k$, the SET-COVER problem asks if there is a sub-collection of $k$ sets $S' \subset S$ whose union covers all the elements of $\mathcal{U}$. That is, $|S'| = k$, and $\bigcup_{S_i \in S'} S_i = \mathcal{U}$. Prove that SET-COVER is NP-complete.

4. (15 pts) We have a set of $n$ transmitters and a set of available frequencies, labeled $\{1, 2, \ldots, m\}$. Each transmitter $i$ is assigned a subset of frequencies $F_i \subset \{1, 2, \ldots, m\}$, and it can only transmit on a frequency in $F_i$. (Different transmitters have different assigned sets.)

In addition, there is a set of interfering transmitter pairs, which cannot transmit at the same frequency because they interfere. For instance, if $(i, j)$ is an interfering pair with $F_i = \{1, 5, 8\}$ and $F_j = \{2, 5, 12, 20\}$, then $i$ and $j$ both cannot be allocated frequency 5. A frequency allocation is valid if each transmitter $i$ gets a frequency from its assigned set $F_i$ and no two interfering transmitters get the same frequency.

Prove that the frequency allocation problem is NP-complete.